CHALMERS

Energetic particle modes: from bump on tail to tokamak

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Outline

- Single particle perspective
- Coherent motion→plasma modes
- Effect of fast particles
- Case study: Bump-on-tail
- Generalisation to 3D world
- Outstanding problems

Single particle perspective

Confinement: a first glance

- Static fields → constant particle energy (E)
- Weak spatial non-uniformity of field→ ”constant” magnetic moment (μ)
- Axisymmetry → constant toroidal angular momentum (p_φ)

$$p_\varphi = mR\mathrm{v}_\varphi + e\psi(r)$$

- Particles have finite excursion from flux surface due to drifts→bounded orbits

Confinement: more detailed

- Axisymmetry is an idealisation, e.g. Ripple effects
- Broken symmetry can lead to loss of confinement
- In general the EM fields are not static...there are many charged particles moving around
- Microscopic time varying fields break invariants of motion and lead to loss of confinement

$$e\phi \ll k_B T$$

- These microscopic fields are the collisions which lead to diffusion of particles out of the tokamak

Coherent motion

Coherent plasma motion

- Coherent motion leading to waves only occurs if the plasma current responds in the same was as the fields, e.g. $E \,\&\, j \sim \sin(\omega t)$
- This only happens if the distribution of particles can be considered as stationary
- Not true in reality, but statistical description, i.e. continuous distribution function, allows this
- Good only when the plasma is sufficiently dense, need many particles per wavelength

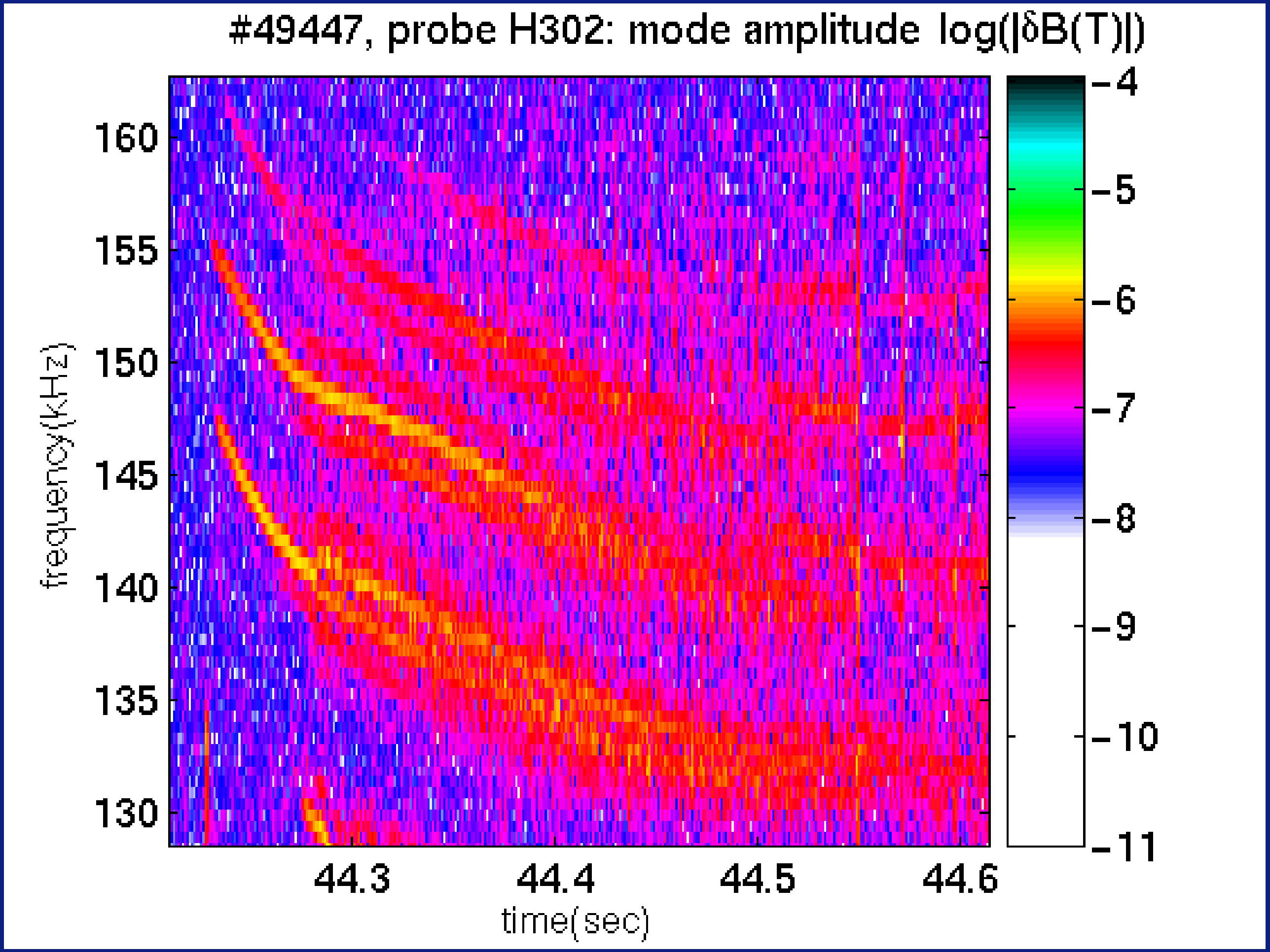
Coherent plasma motion

- Waves need energy
- Tokamak not in thermodynamic equilibrium

$$\nabla P = j \times B$$

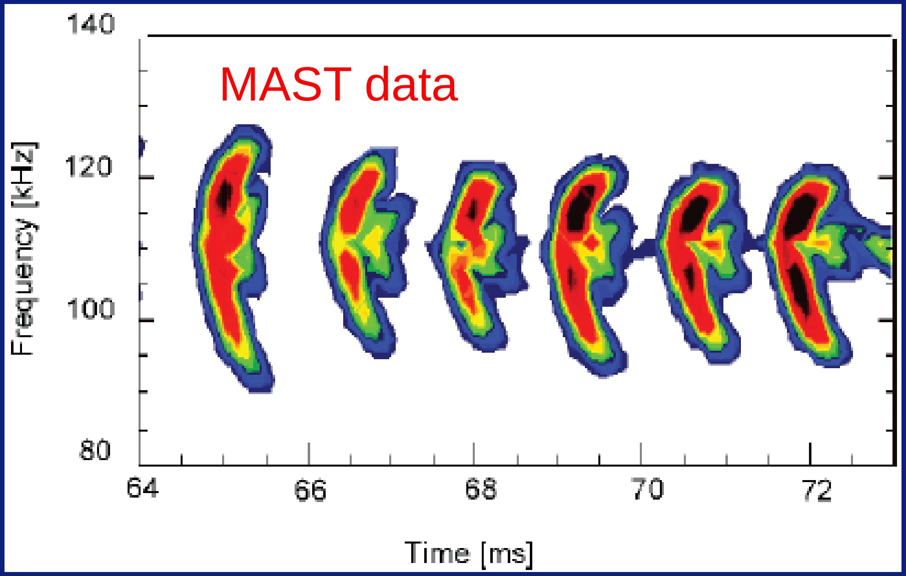
- Current and density gradient drive waves, e.g. Kink, ballooning modes, typically low frequency
- Another source of free energy in fast particles – The waves are characterised by bulk plasma but are excited by the low density fast population

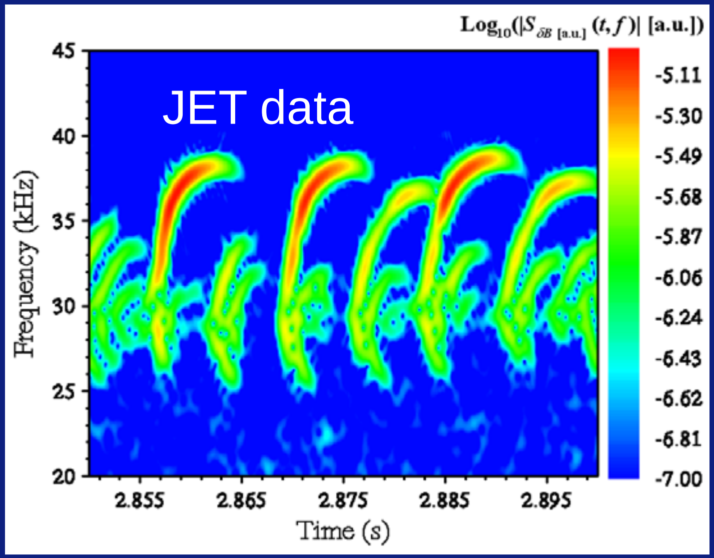
Fast particle driven modes – Soft nonlinearity – TAEs via ICRH on JET

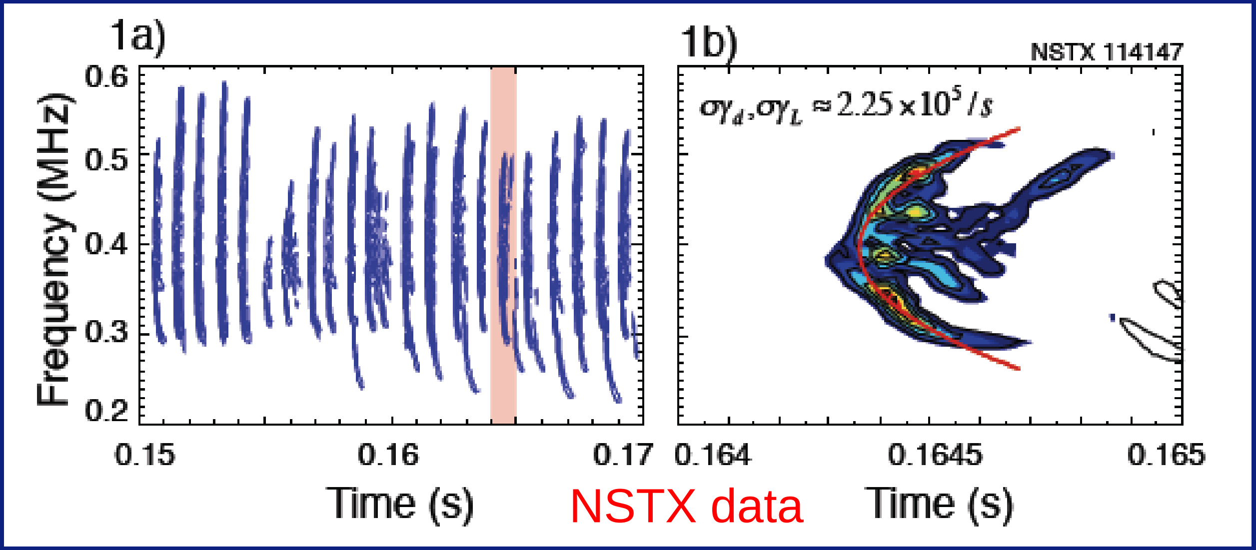


Heeter et.al PRL 85, 3177 (2000)

Fast particle driven modes – Rapid sweeping

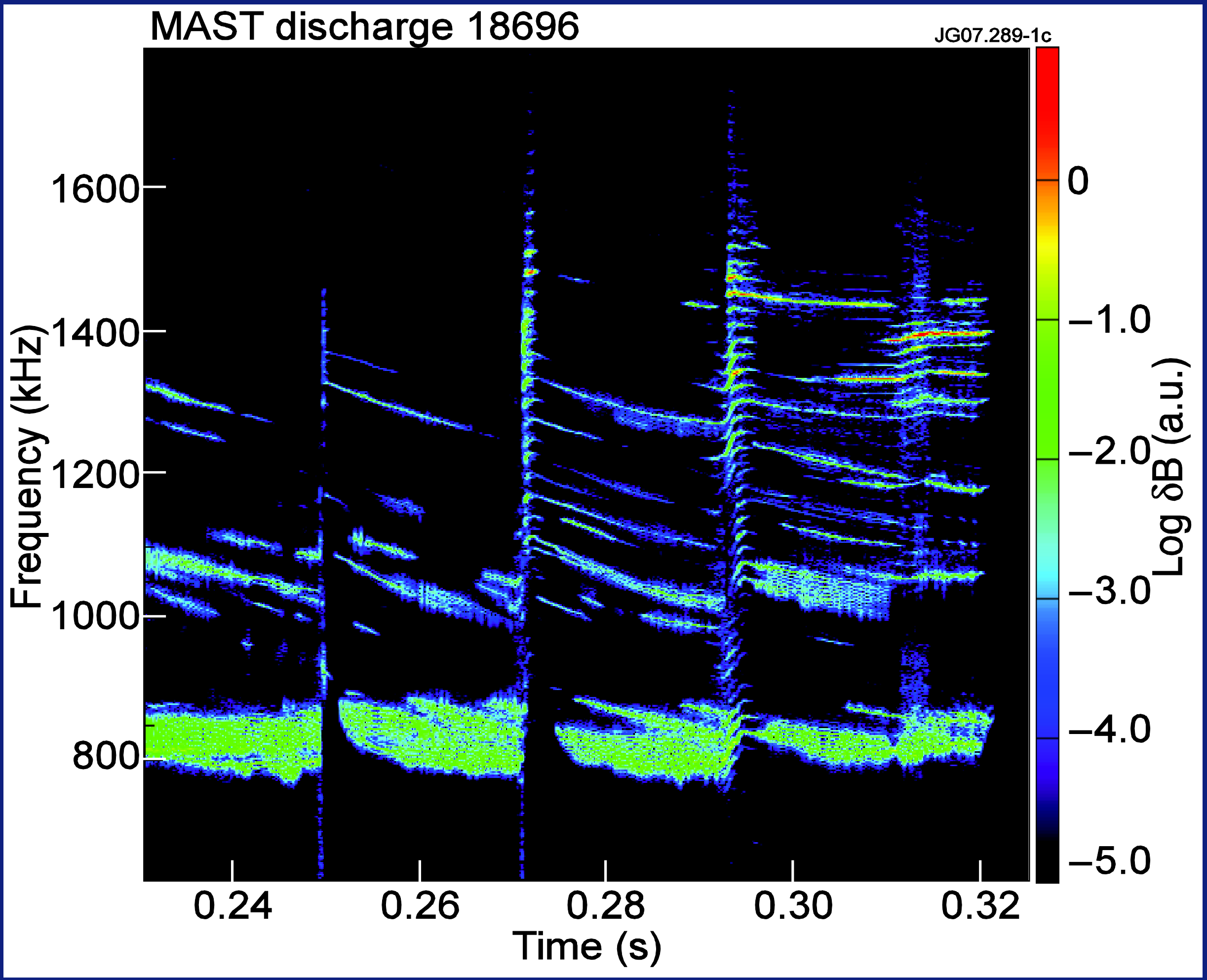






The ms timescale of these events is much shorter than the energy confinement time in the plasma

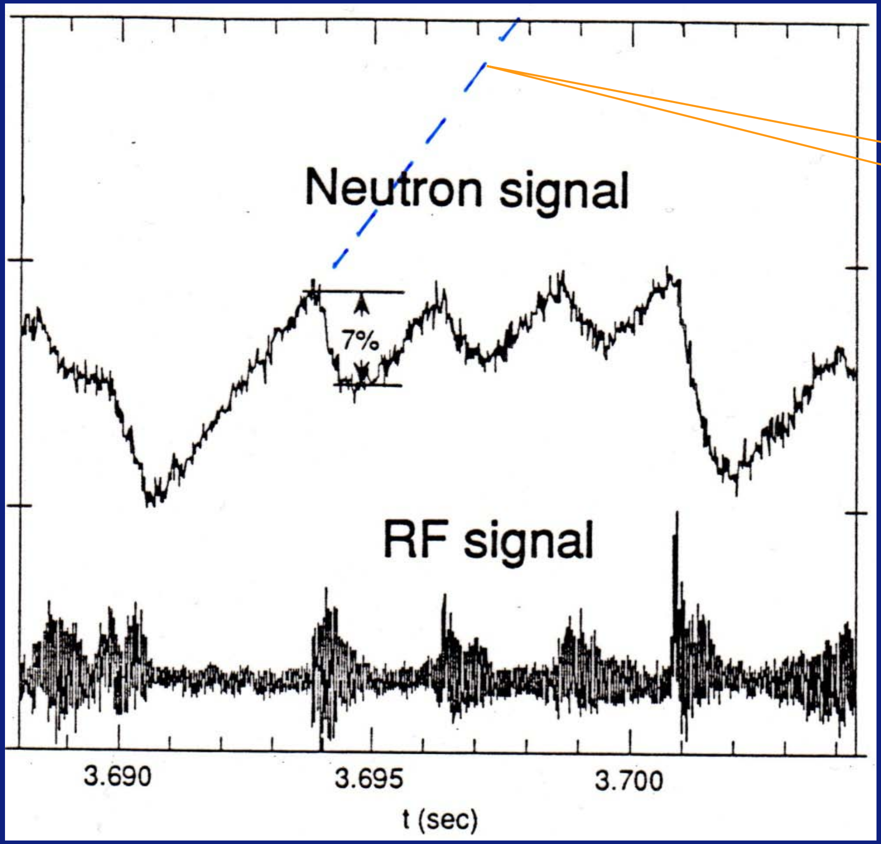
Fast particle driven modes – Mixed – Beam driven CAEs on MAST



Gryaznevich et.al Nucl. Fus. 48, 084003 (2004)

Fast particle driven modes – Particle loss in TFTR

Saturation of the neutron signal reflects anomalous losses of the injected beams. The losses result from Alfvénic activity.



Projected growth of the neutron signal

K. L. Wong et.al PRL 66, 1874 (1991)

Coherent plasma motion

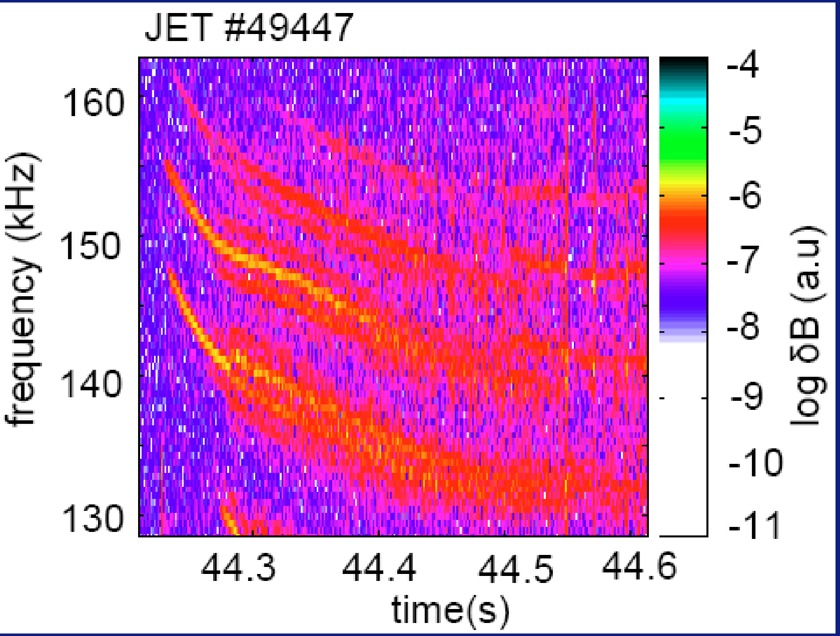
- Coherent motion of plasma can have a much larger effect than collisions
- Effect of waves on confinement of particles cannot be universally predicted
- Each case must be dealt with separately
- We will focus on instabilities driven by fast particles in this lecture

Effect of fast particles on waves

The Questions

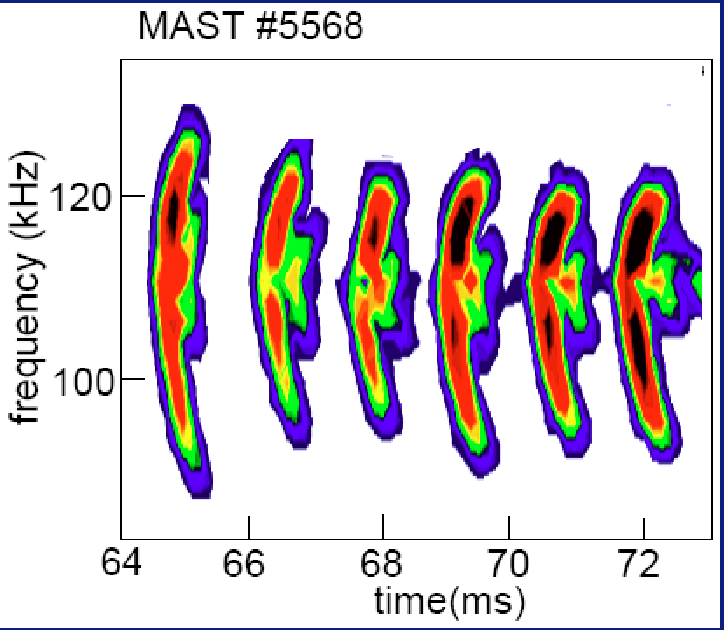
- How does a low density population produce a large effect
- How do the fast particles produce such rich non linear evolution at different timescales
- How is it that the same modes driven by different particles look so different

ICRH drive (TAE-JET)



Heeter et.al PRL 85, 3177 (2000)

NBI drive (TAE-MAST)



Pinches et.al PPCF, 46, S47 (2004)

Resonance

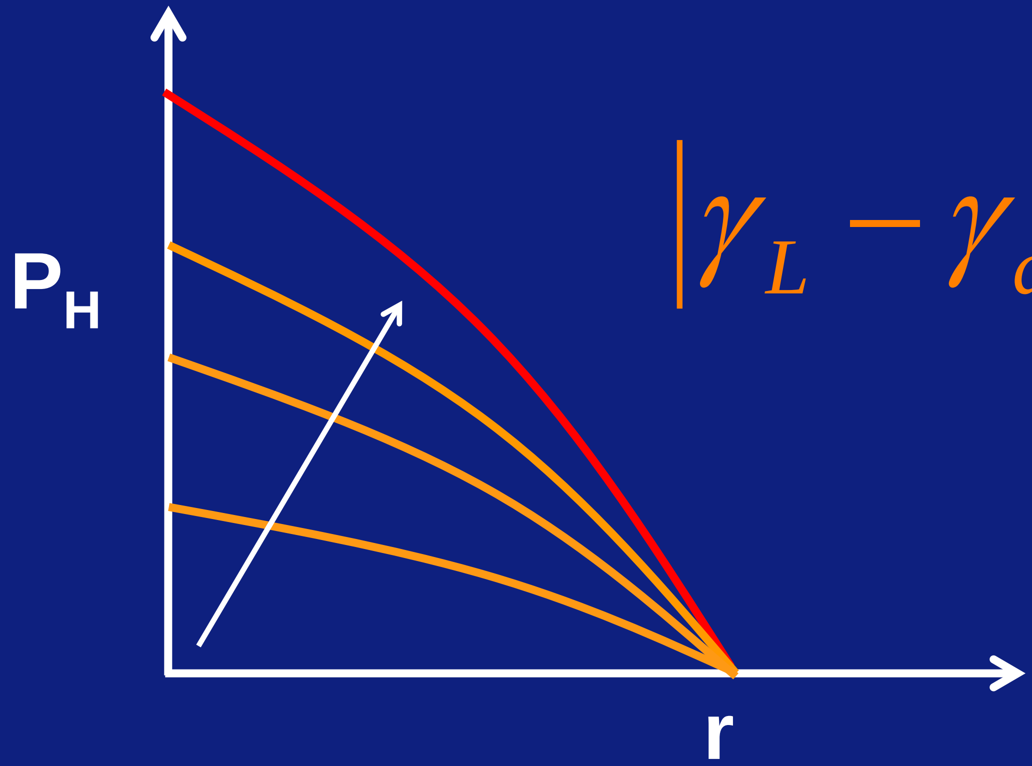
- Special group of particles that strongly interact with a wave

$$\text{force} \sim e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \xrightarrow{1D} e^{i(k(x_0+\mathrm{v}_0 t)-\omega t)}$$

- $v_0=\omega/k$ gives non oscillating force on particle
- Provides a channel for energy to go from the source into the coherent motion of background not thermal motion
- Allows low density fast particles to pump/drive the wave

Marginal stability

- System evolves through a threshold



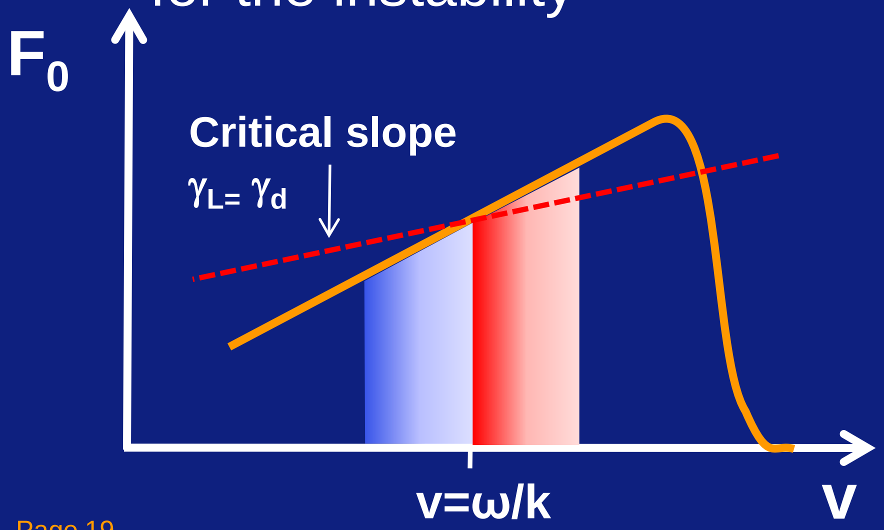
$$\left|\gamma_L - \gamma_d\right| \ll \gamma_L, \gamma_d$$

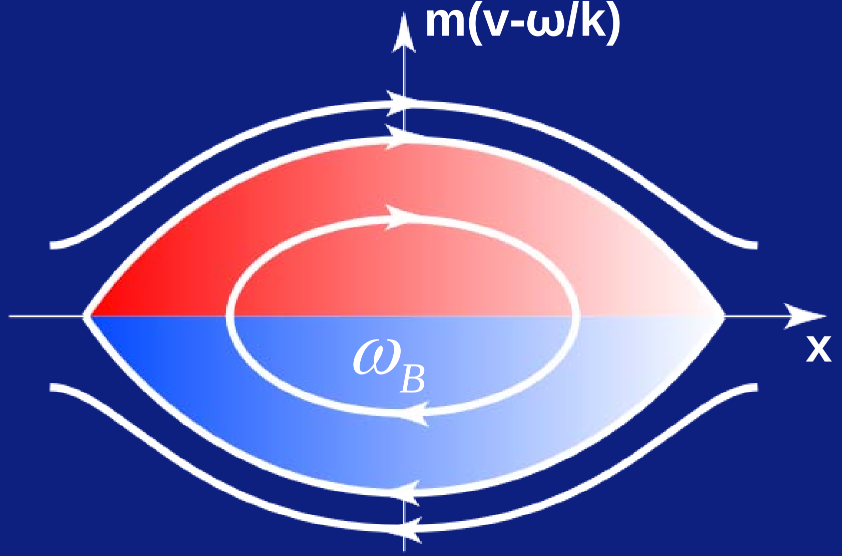
- Collision times are comparable to growth times

Case study: Bump-on-tail

Bump on tail - Basic ingredients

- Particle injection and effective collisions, ν_{eff}, create an inverted distribution of energetic particles $F_0(v)$
- Discrete spectrum of unstable electrostatic modes
- Instability drive, $\gamma_L \sim dF_0/dv$, due to wave-particle resonance (ω-$kv=0$)
- Background dissipation rate, γ_d, determines the critical gradient for the instability



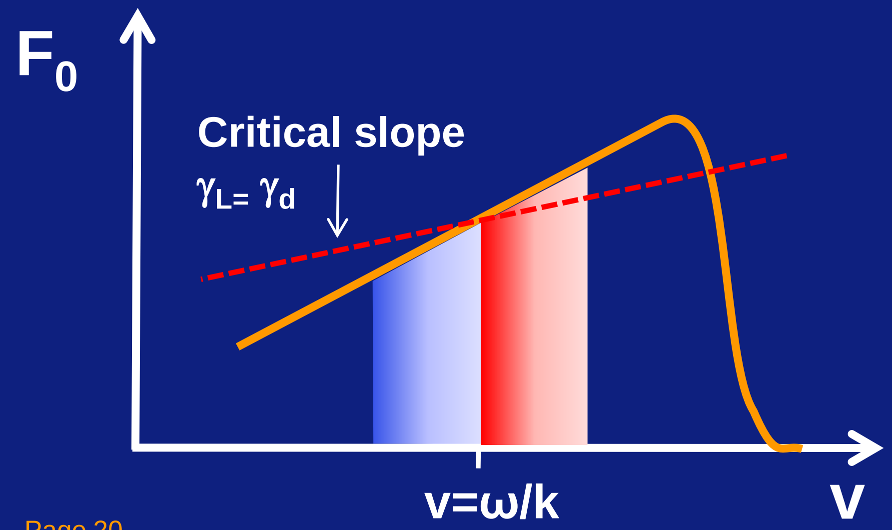


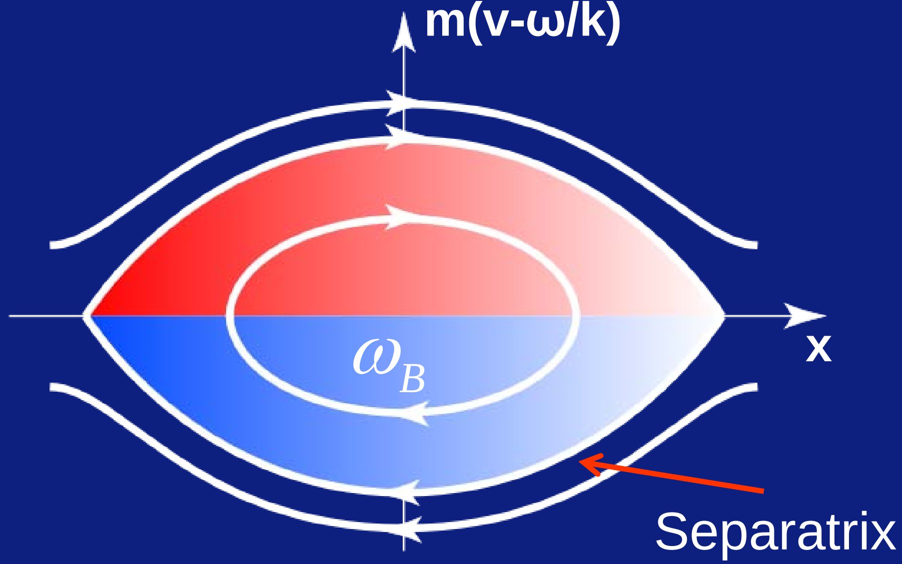
Bump on tail - Basic ingredients

- In wave frame the electric potential creates trapped and passing particles

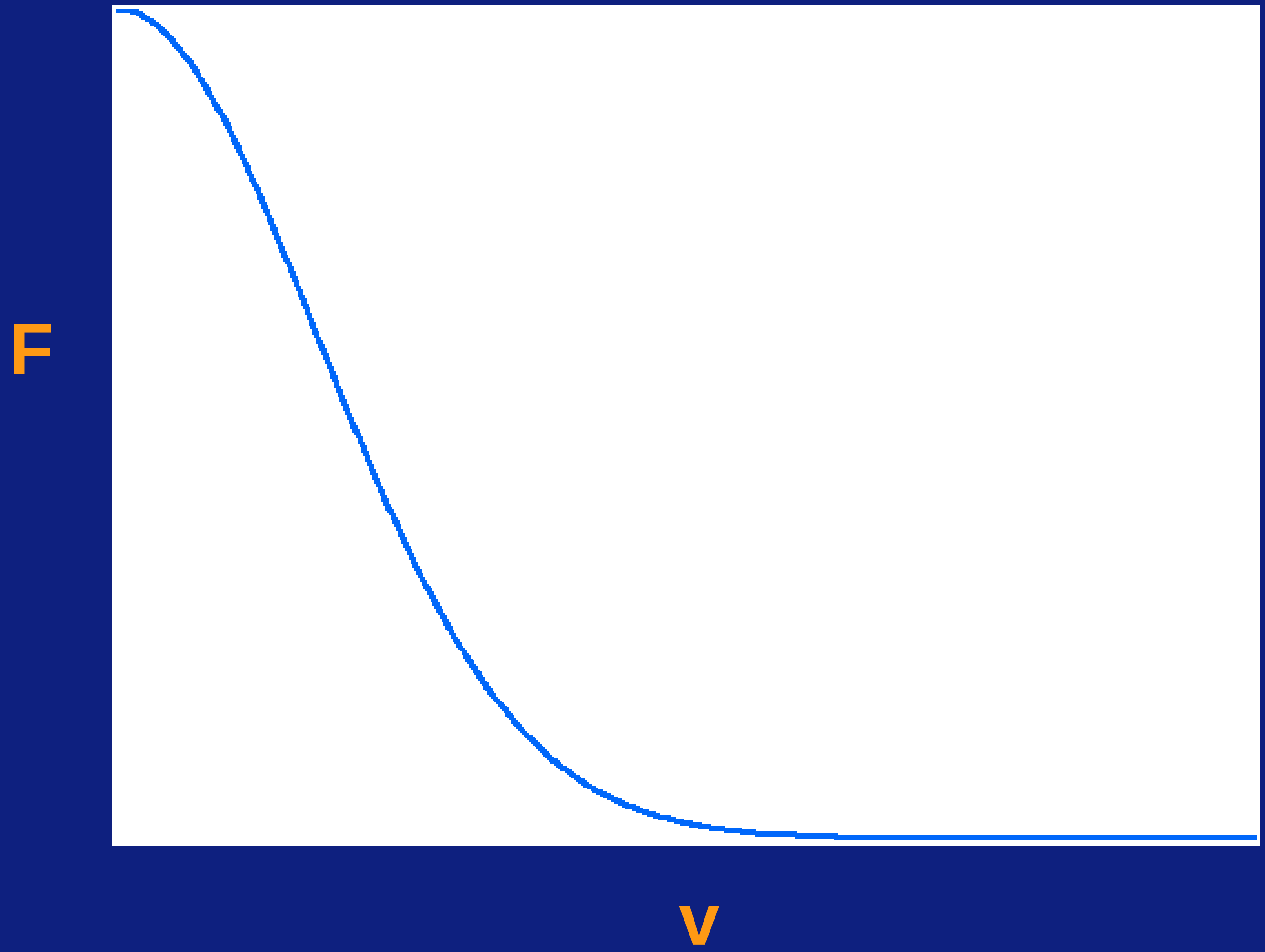
$$\mathcal{E} = \frac{1}{2} m\mathrm{v}^2 - |e|\phi \rightarrow \mathrm{v}_{sep} = \sqrt{2/m}\sqrt{\phi_{max} - \phi}$$

- Separatrix is the trapped/passing boundary
- Motion in phase space is that of a pendulum with frequency ω_B determined by amplitude of field $\omega_B \sim \hat{E}^{1/2}$





No dissipation or collisions – Schematic



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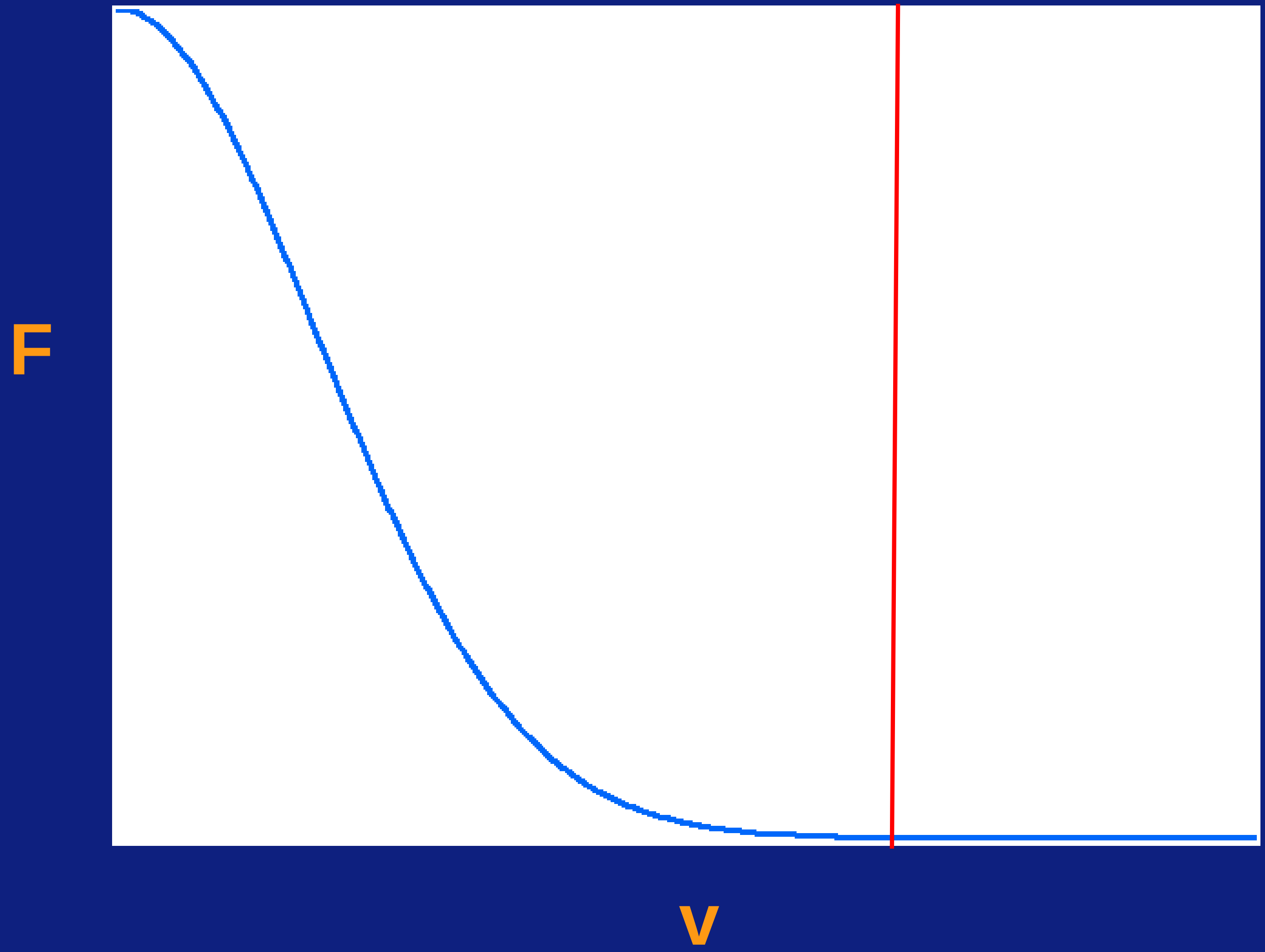

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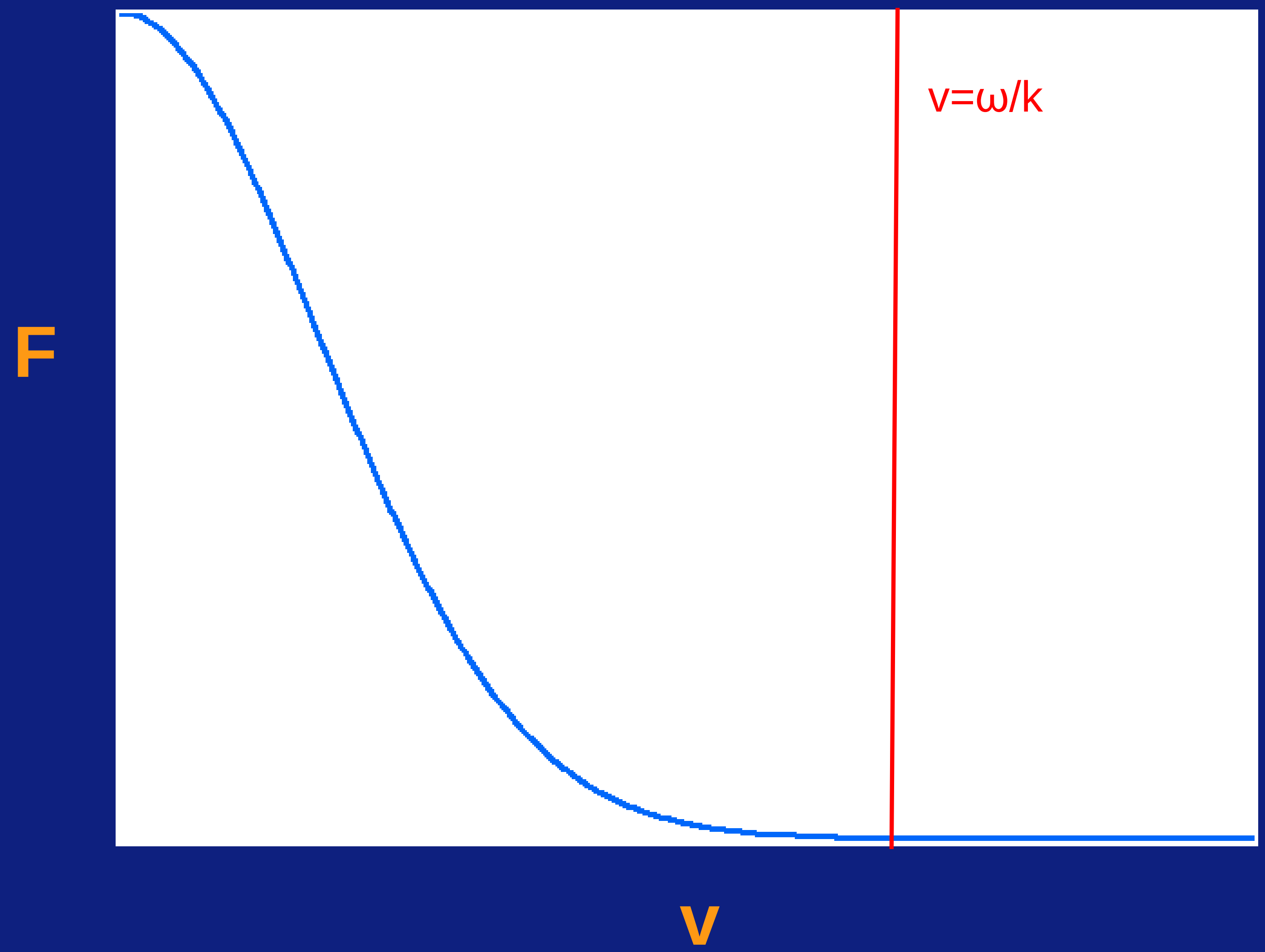

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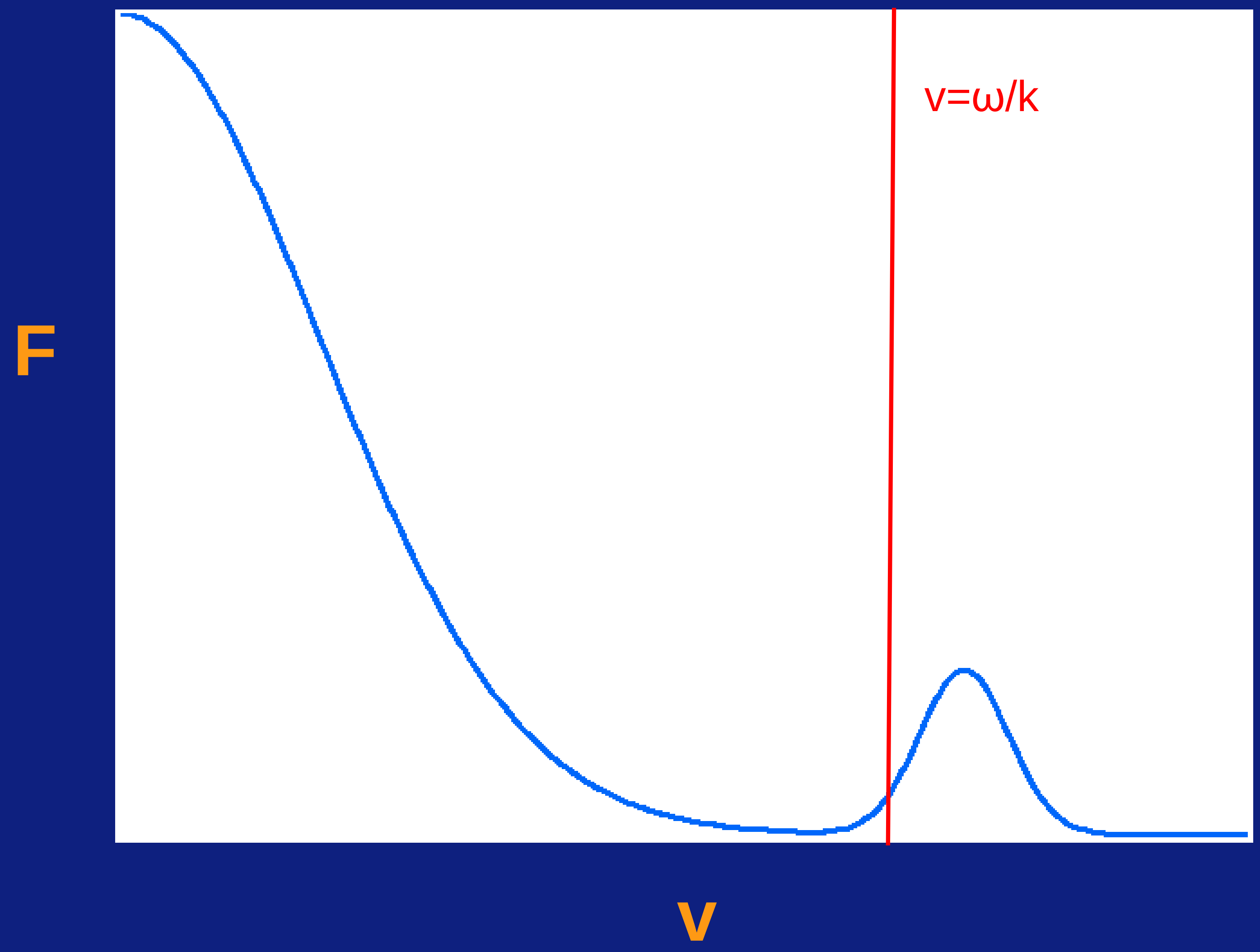

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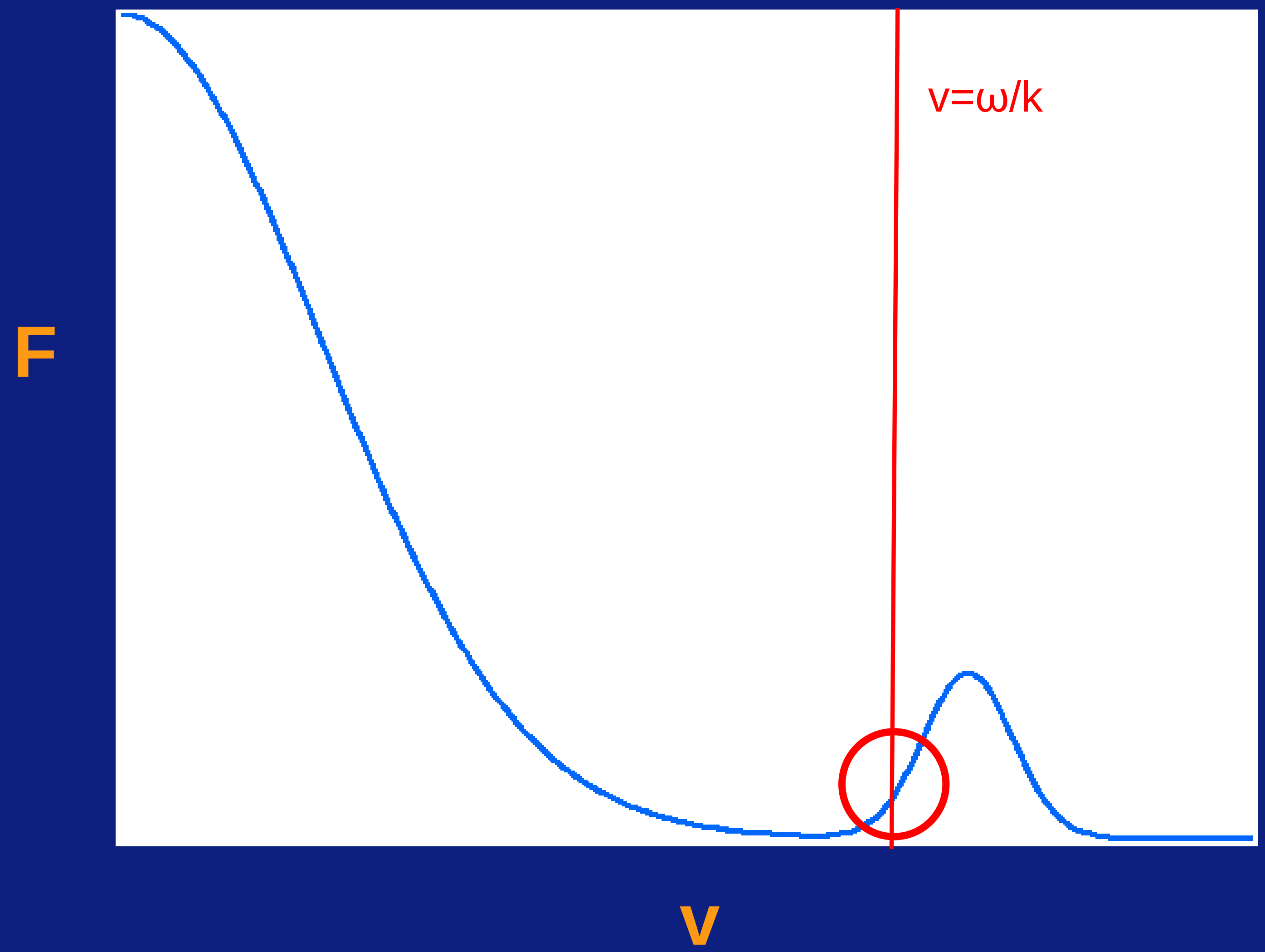

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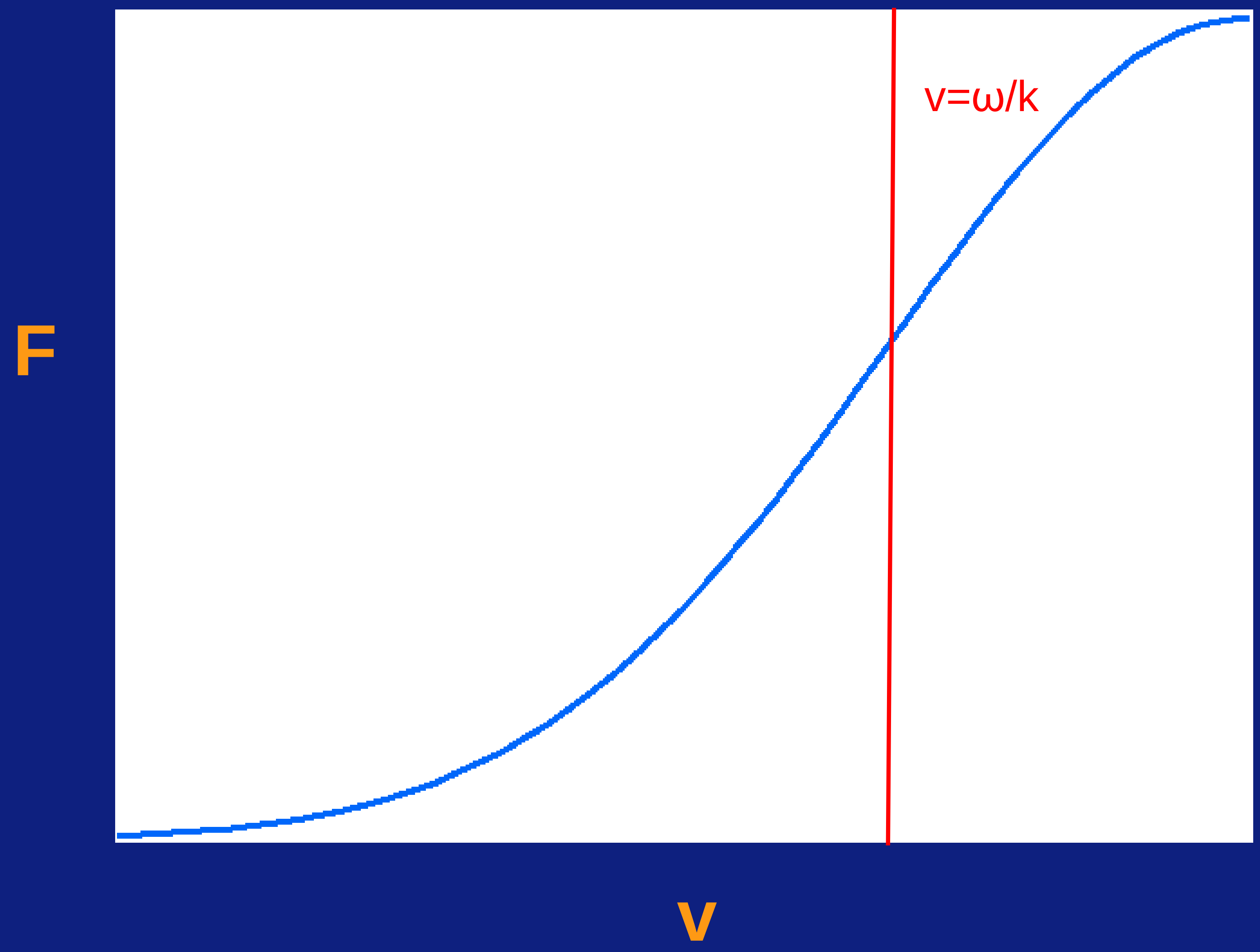

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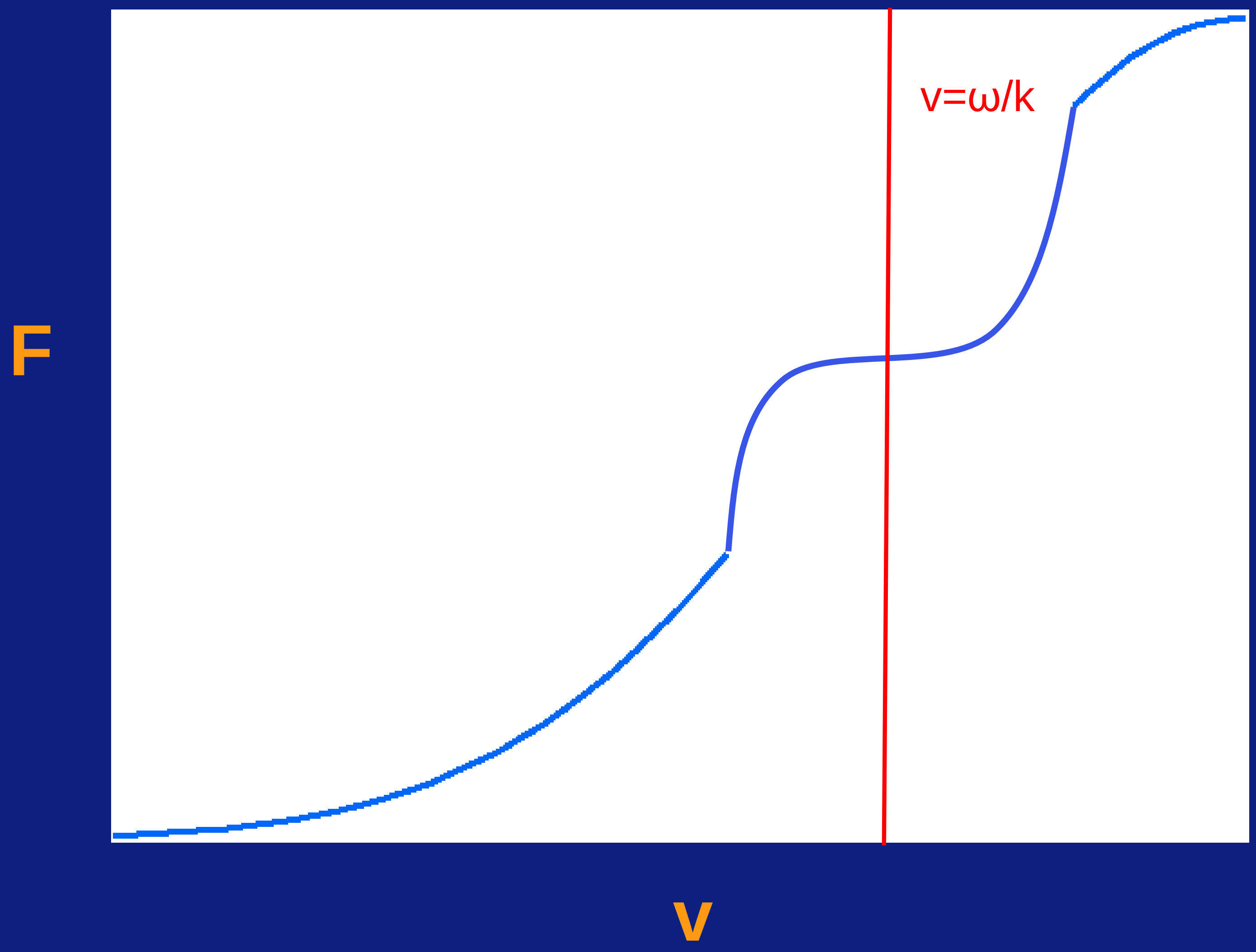
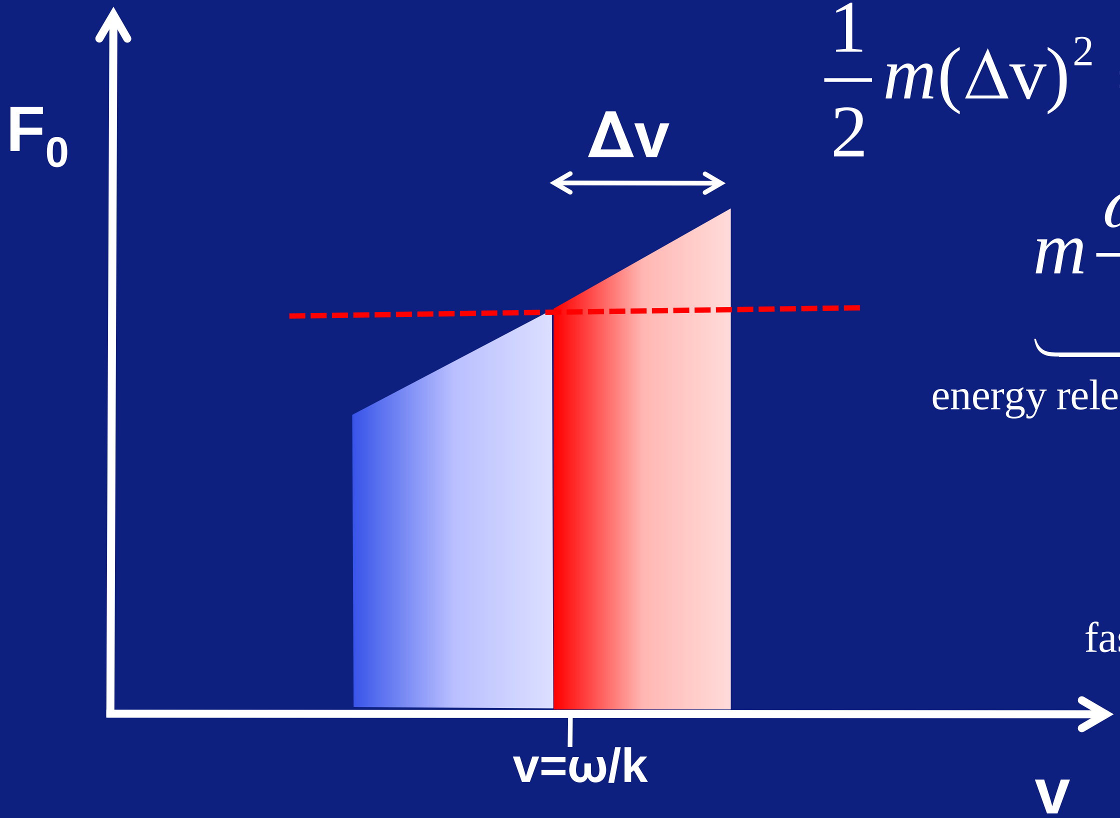


No dissipation or collisions – Saturation level



$$\frac{1}{2}m(\Delta \mathrm{v})^2 = -|e|\phi \quad \rightarrow \quad \Delta \mathrm{v} \sim \sqrt{\hat{E}}$$

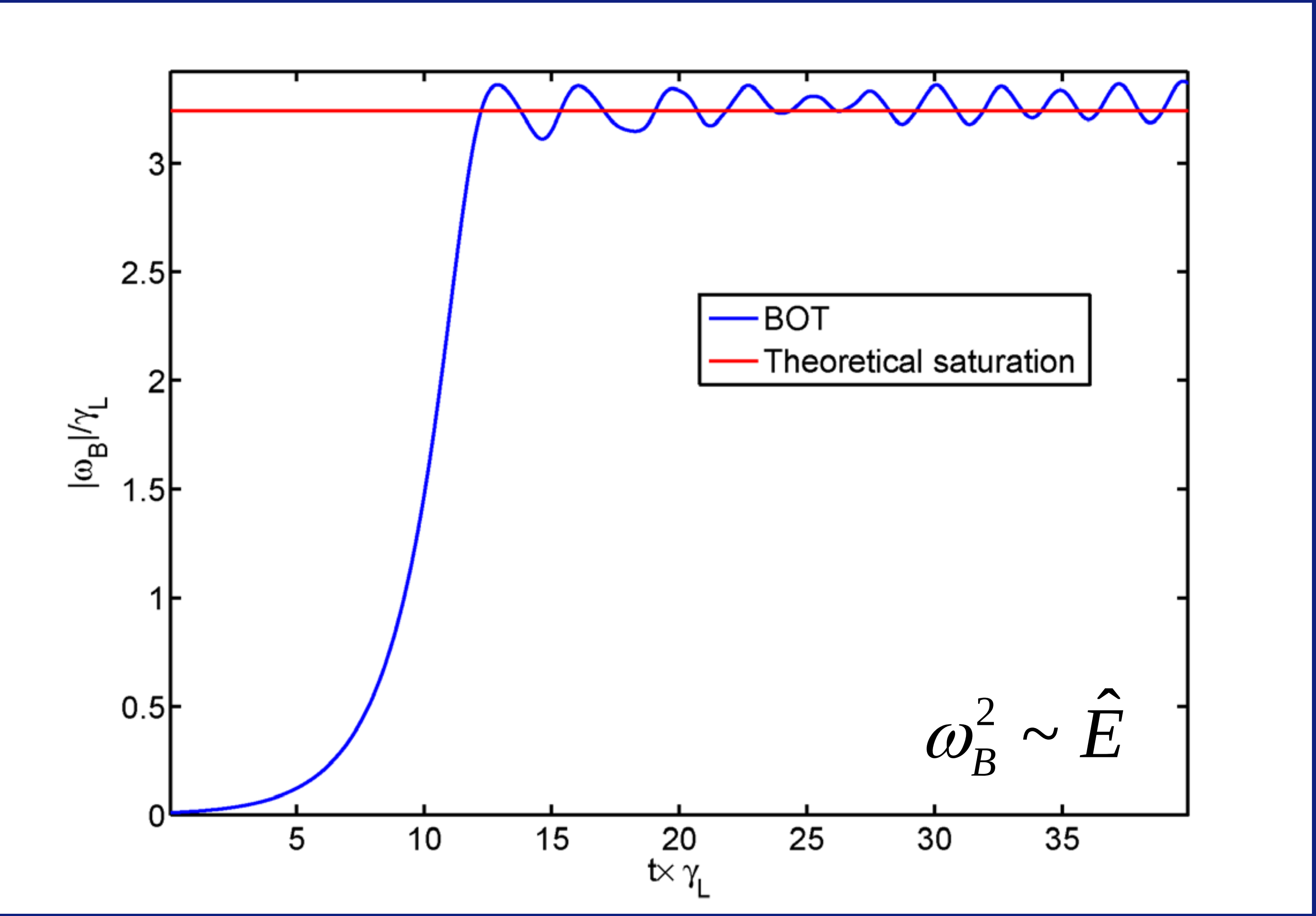
$$\underbrace{m\frac{\omega}{k}\Delta \mathrm{v}}_{\text{energy release per particle}} \times \underbrace{\frac{1}{2}\Delta \mathrm{v}\frac{\partial F_0}{\partial \mathrm{v}}\Delta \mathrm{v}}_{\text{no. particles}}$$

$$\underbrace{\hat{E}^{3/2}}_{\text{fast particles energy}} \sim \underbrace{\hat{E}^{2}}_{\text{wave energy}}$$

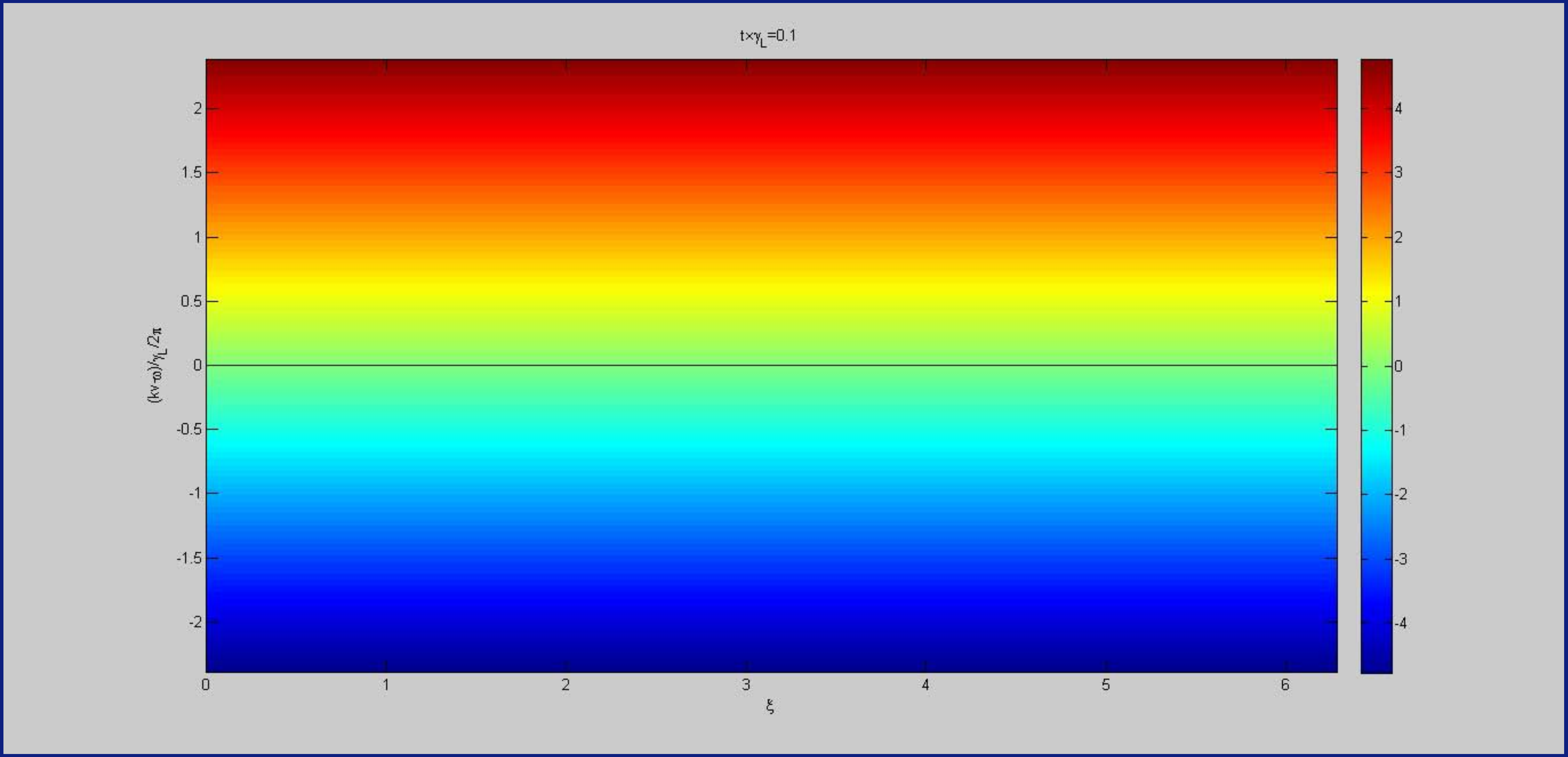
- Wave grows until fast particle energy release cannot support the wave energy

$$\rightarrow \omega_B \equiv \sqrt{\frac{|e|k\hat{E}}{m}} \sim \gamma_L$$

No dissipation or collisions – Saturation level



No dissipation or collisions – Phase space plateaux



- Distribution is only strongly perturbed inside separatrix (black line)

Bump on tail – Key bits of physics

- Wave creates perturbations in velocity space around resonance
- Mixing area is bounded by the separatrix, determined by bounce frequency. This separates trapped and passing particles
- Bounce frequency is on the order of γ_L
- γ_L is small compared to wave frequency, which means the electric field is ”small” so that perturbation of background is small which means linear

Bump on tail - formalism

- Linear cold background with sinusoidal field $E = \frac{1}{2}\left[\hat{E}(t)e^{i\zeta} + \text{c.c.}\right]$

$$\frac{\partial V}{\partial t} = -\frac{|e|E}{m} - \nu_c V$$

$$\zeta \equiv kx - \omega t$$
$$u \equiv k\text{v} - \omega$$

- Kinetic fast particle population $F = F_0 + f_0 + \sum_{n=1}^{\infty}\left[f_n \exp(in\zeta) + c.c.\right]$

$$\frac{\partial F}{\partial t} + u\frac{\partial F}{\partial \zeta} - \frac{|e|k}{2m}\left[\hat{E}(t)e^{i\zeta} + \text{c.c.}\right]\frac{\partial F}{\partial u} = \left.\frac{dF}{dt}\right|_{\text{coll}}$$

- Current from cold background obtained perturbatively using smallness of wave growth and dissipation

$$\frac{\partial \hat{E}}{\partial t} = -\frac{\omega}{\varepsilon_0 k^2} e\int f_1 du - \gamma_d \hat{E}$$

$$\gamma_d = \nu_c / 2$$

Collisionality – First glance

- Marginal stability allows collisions to compete with mode growth

$$\left|\gamma_L - \gamma_d\right| \sim \nu_{\mathrm{eff}}$$

- Krook and diffusion have been studied

$$\left.\frac{dF}{dt}\right|_{\mathrm{coll}} = \beta\left(F - F_0\right) \qquad \left.\frac{dF}{dt}\right|_{\mathrm{coll}} = \frac{\nu^3}{k^2}\left(\frac{\partial^2 F}{\partial \mathrm{v}^2} - \frac{\partial^2 F_0}{\partial \mathrm{v}^2}\right)$$

- Note: Krook is normally to mock up diffusion, but can actually be physical if collisions move particle immediately out of resonance (not typical in fusion conditions)

$$\nu_{\mathrm{eff}} = \max\left\{\beta, \nu\right\}$$

Near threshold ordering

- Perturbative approach applied → time scales shorter than non-linear bounce period of the wave ω_B^{-1}

$$\omega_B^2 = ek\hat{E} / m$$

- Can be maintained indefinitely if collision frequency is much larger than bounce frequency

- The distribution function will not be significantly perturbed:

$$F_0 \gg f_1 \gg f_0, f_2$$

Near threshold ordering

$$\frac{\partial f_0}{\partial t} - \nu^3 \frac{\partial^2 f_0}{\partial u^2} + \beta f_0 = -\frac{ek}{2m}\left(\hat{E} \frac{\partial f_1^*}{\partial u} + \text{c.c} \right)$$

$$\frac{\partial f_1}{\partial t} + iuf_1 - \nu^3 \frac{\partial^2 f_1}{\partial u^2} + \beta f_1 = -\frac{ek}{2m} \hat{E} \frac{\partial}{\partial u}\left(F_0 + f_0 + f_2 \right)$$

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$$f_1 \sim$$

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$$f_1 \sim \gamma_L \hat{E}$$

Near threshold ordering

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$$f_1 \sim \gamma_L \hat{E} + \gamma_L c_3 \hat{E}^3 + \dots$$

Near threshold ordering

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$$f_1 \sim \gamma_L \hat{E} + \gamma_L c_3 \hat{E}^3 + \ldots$$

$$\frac{\partial \hat{E}}{\partial t} = -4 \frac{\omega}{k^2} \pi e \int f_1 du - \gamma_d \hat{E}$$

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$$f_1 \sim \gamma_L \hat{E} + \gamma_L c_3 \hat{E}^3 + \ldots$$

$$\frac{\partial \hat{E}}{\partial t} \sim \gamma_L \hat{E}\left(1 + c_3 \hat{E}^2 + \ldots\right) - \gamma_d \hat{E}$$

Mode evolution equation - sign of cubic nonlinearity

- First term leads to exponential growth, we must have a negative second term to have saturation.

$$\frac{dA}{d\tau} = A(\tau) - \frac{1}{2}\int_0^{\tau/2} dz\, z^2 A(\tau - z) \int_0^{\tau-2z} dx\, e^{-\hat{\nu}^3 z^2 (2z/3+x) - \hat{\beta}(2z+x)} \times$$

$$A(\tau - z - x) A^*(\tau - 2z - x)$$

$\hat{\nu}$ - Diffusion coefficient

$\hat{\beta}$ - Krook coefficient

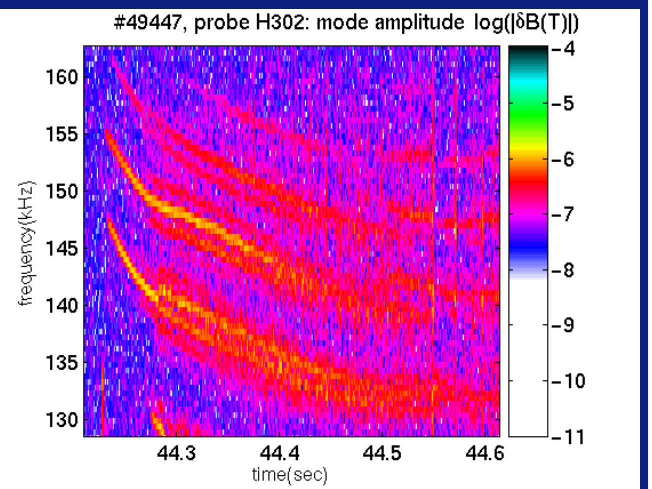
- Minus sign must persist for steady state
- For Krook and diffusion – Sign can only flip for low collisionality

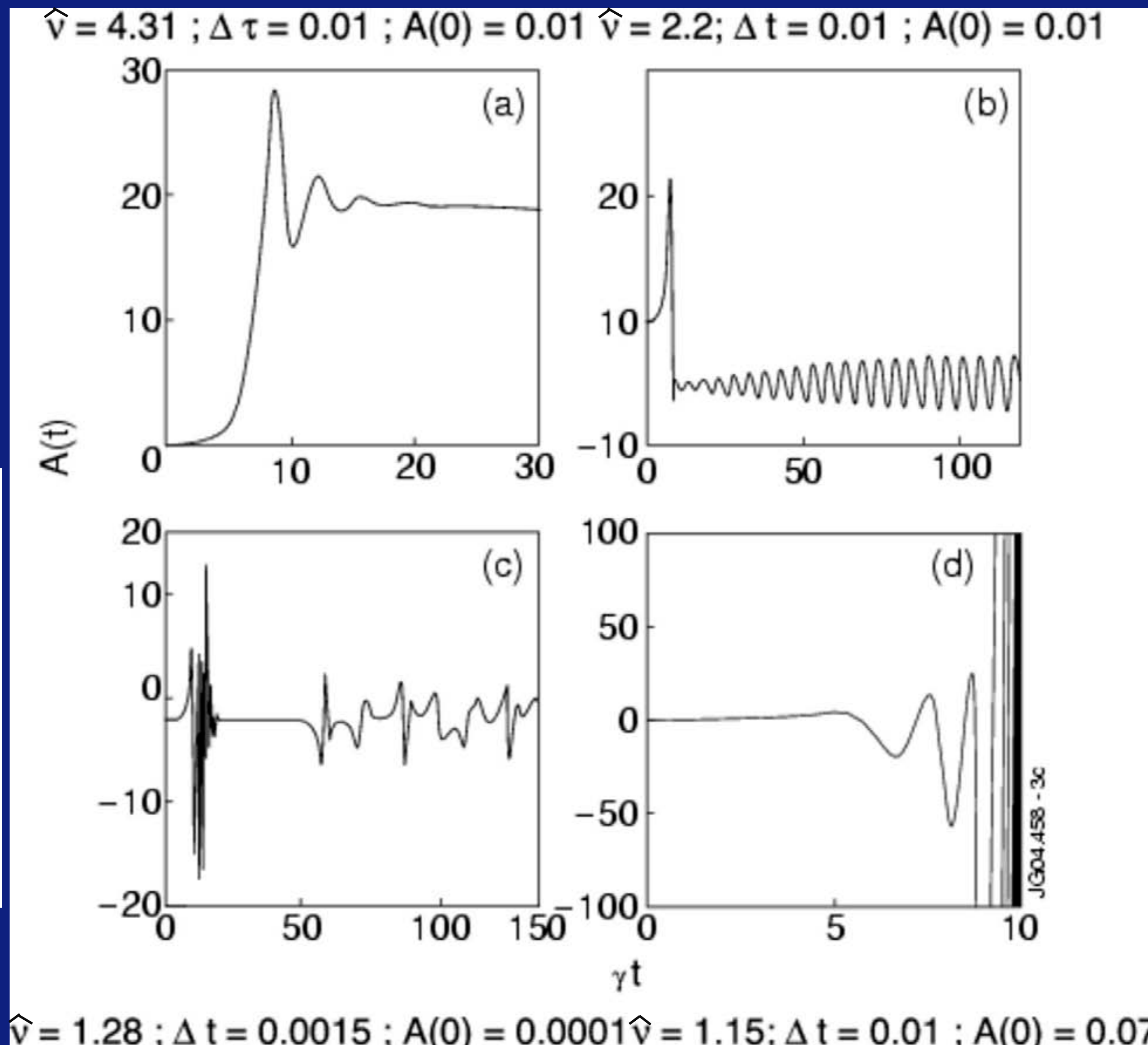
Berk et.al PRL, 76, 1256 (1996)

Breizman et.al PoP, 4, 1559 (1997)

Collisionality affects mode saturation

$$\hat{\nu} = \frac{\nu}{\gamma_L - \gamma_d}$$





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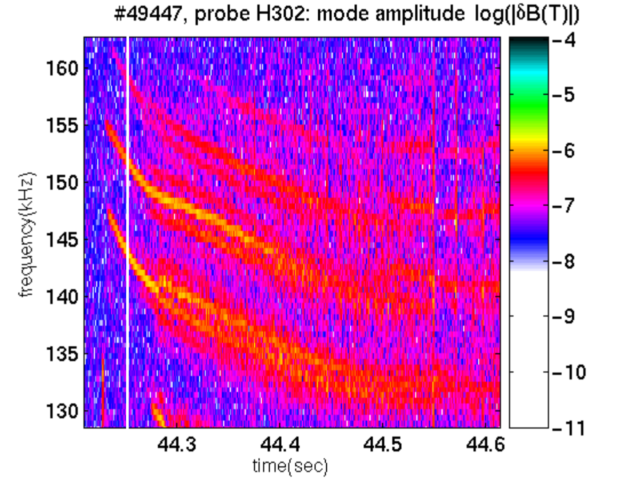
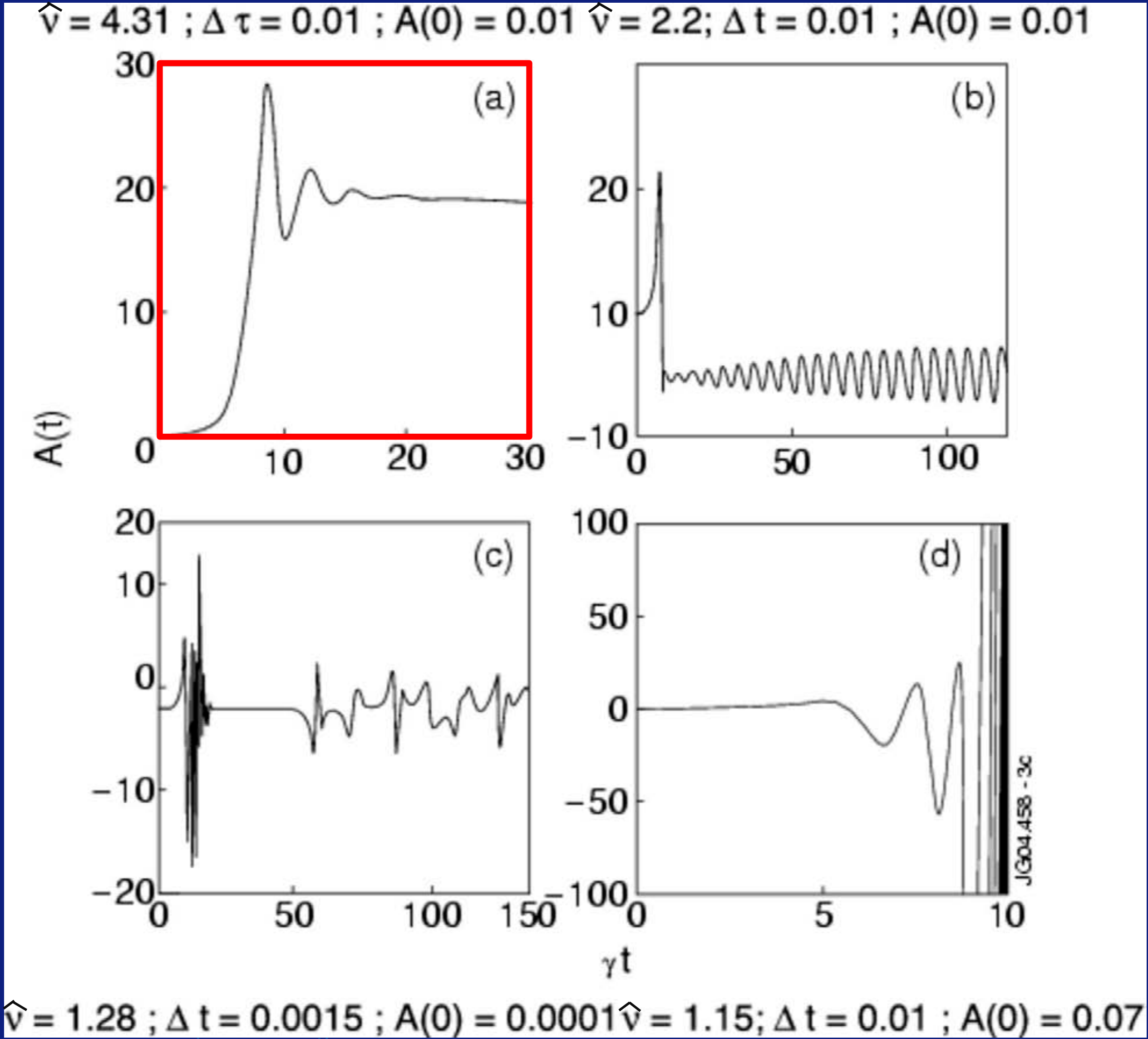
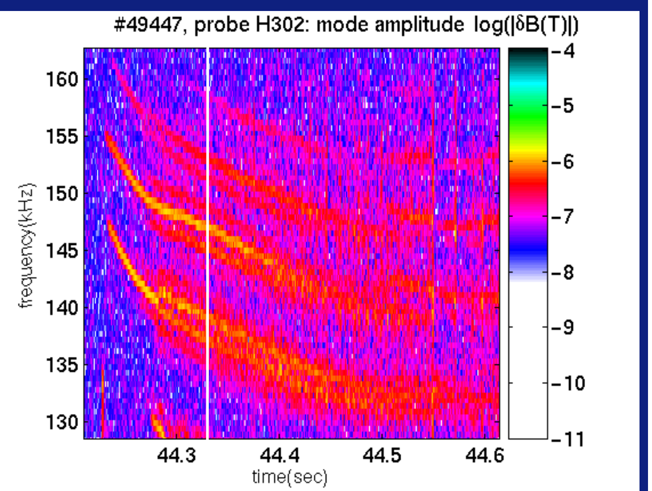


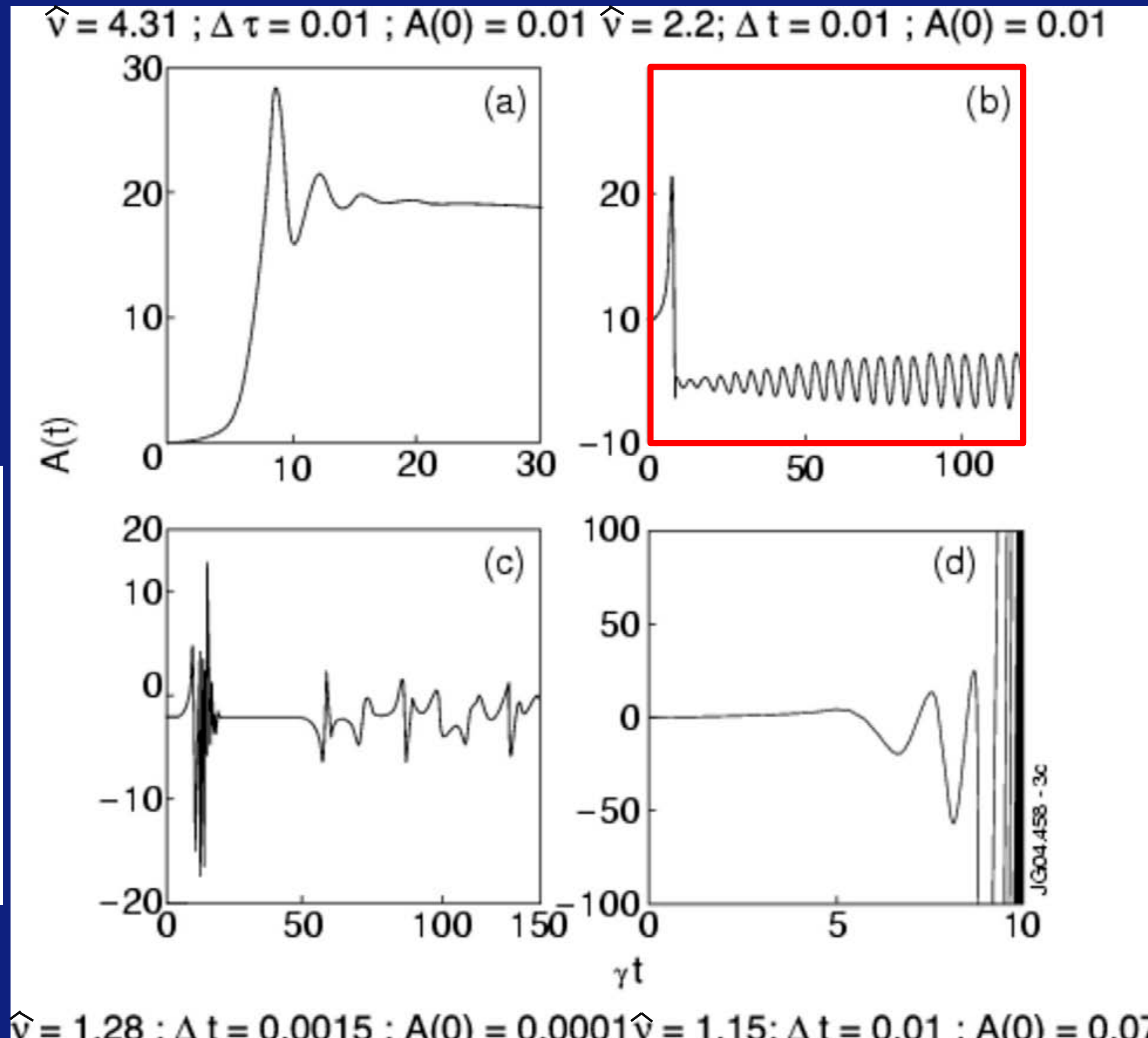


Heeter et.al PRL, 85, 3177 (2000)

Collisionality affects mode saturation

$$\hat{\nu} = \frac{\nu}{\gamma_L - \gamma_d}$$





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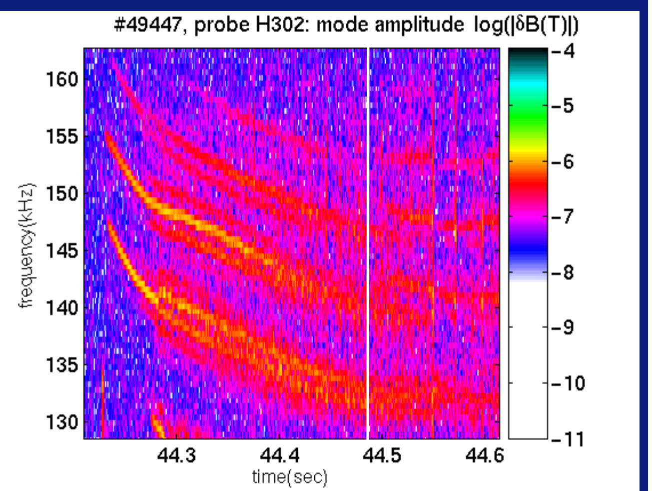

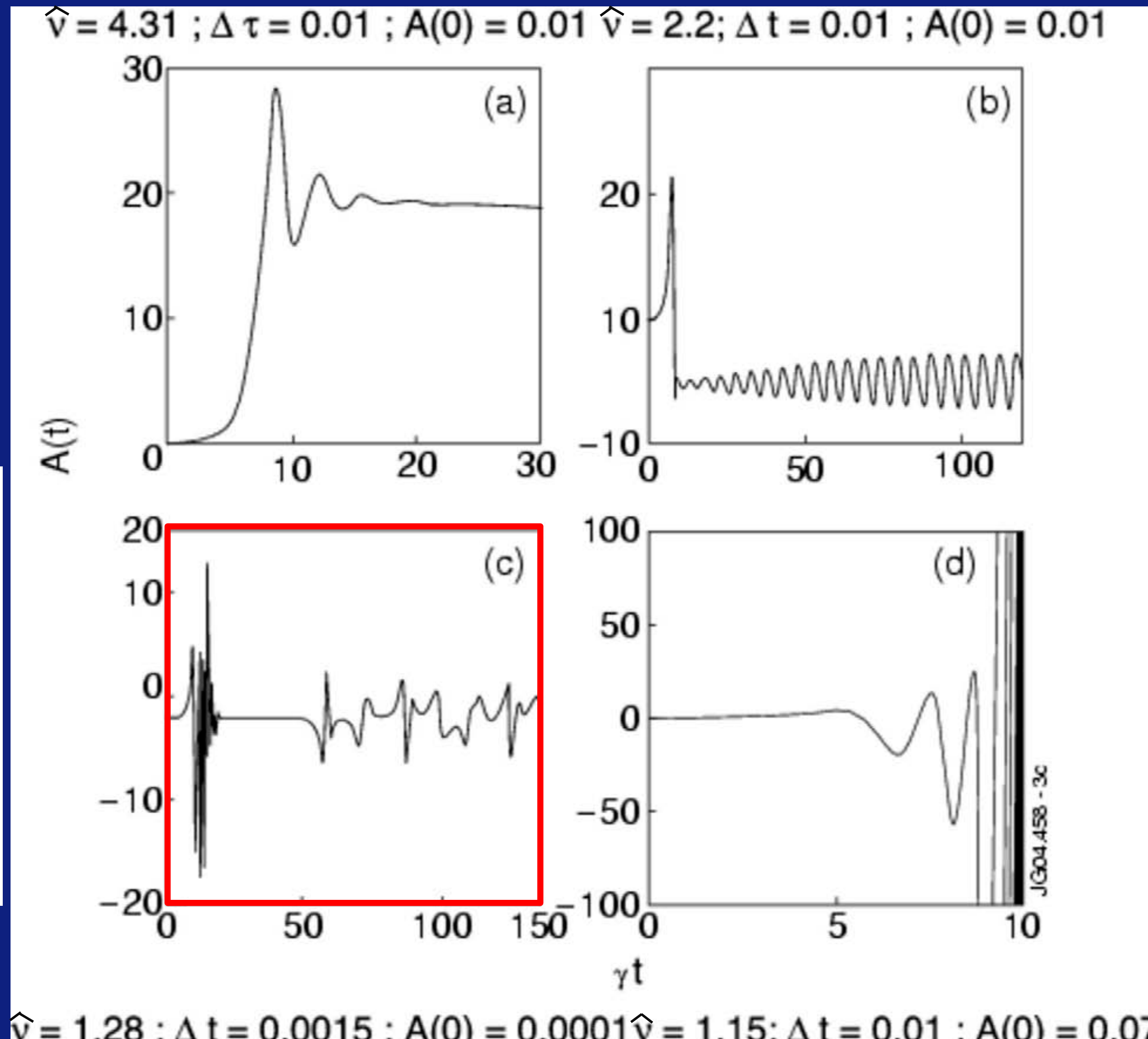

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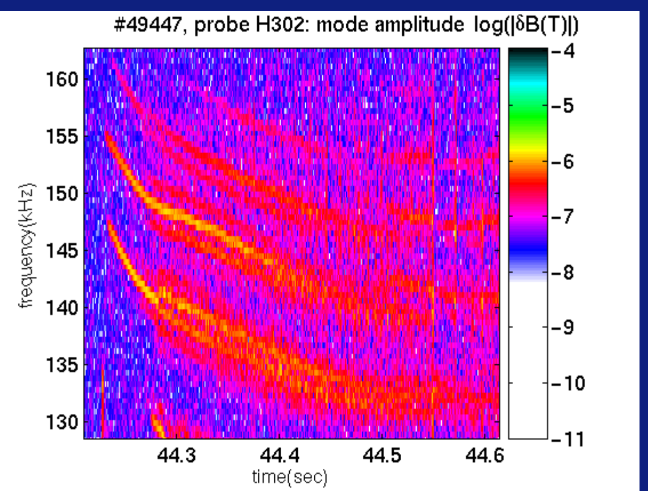
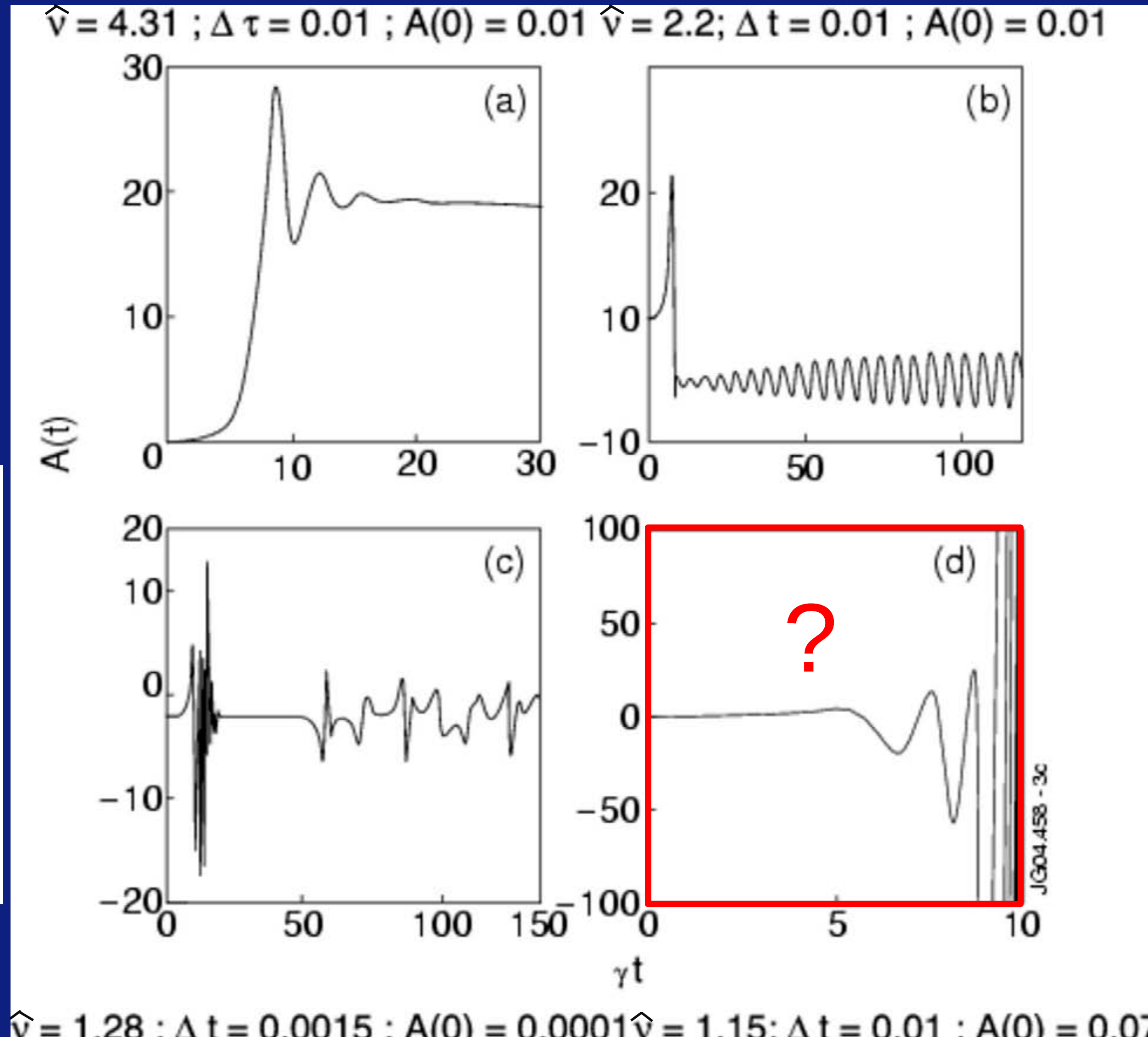




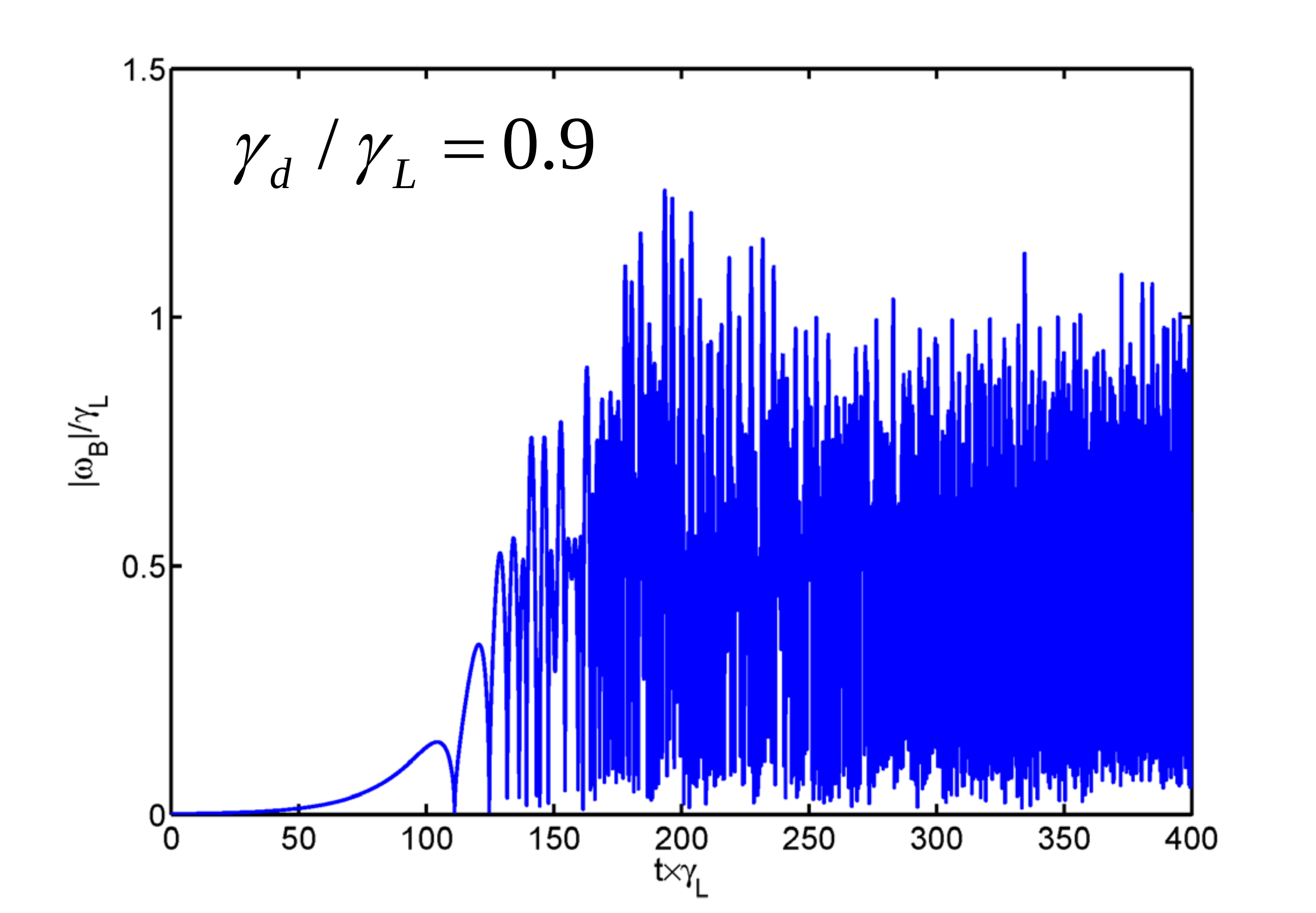
Heeter et.al PRL, 85, 3177 (2000)

Collisionality affects mode saturation

- This was only a perturbative analysis (cubic order in E)
- Fully non-linear treatment requires numerical techniques
- Techniques should take advantage of separation of times scales $(\gamma \ll \omega)$ i.e. Use BOT code: Fourier space code that runs in a couple of minutes on a laptop
- What happens in the explosive regime?

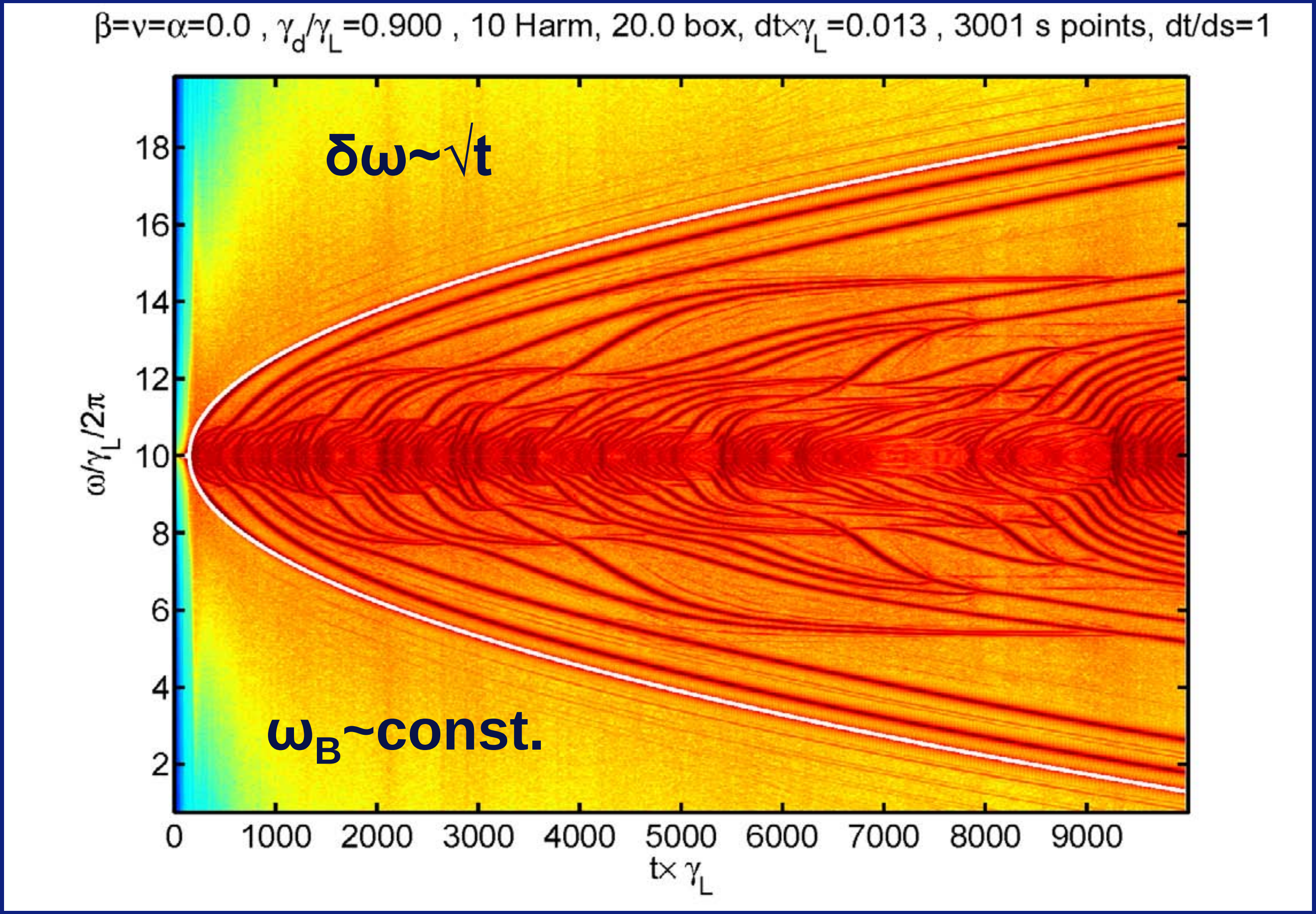
BREAK!

Marginal stability – No saturation

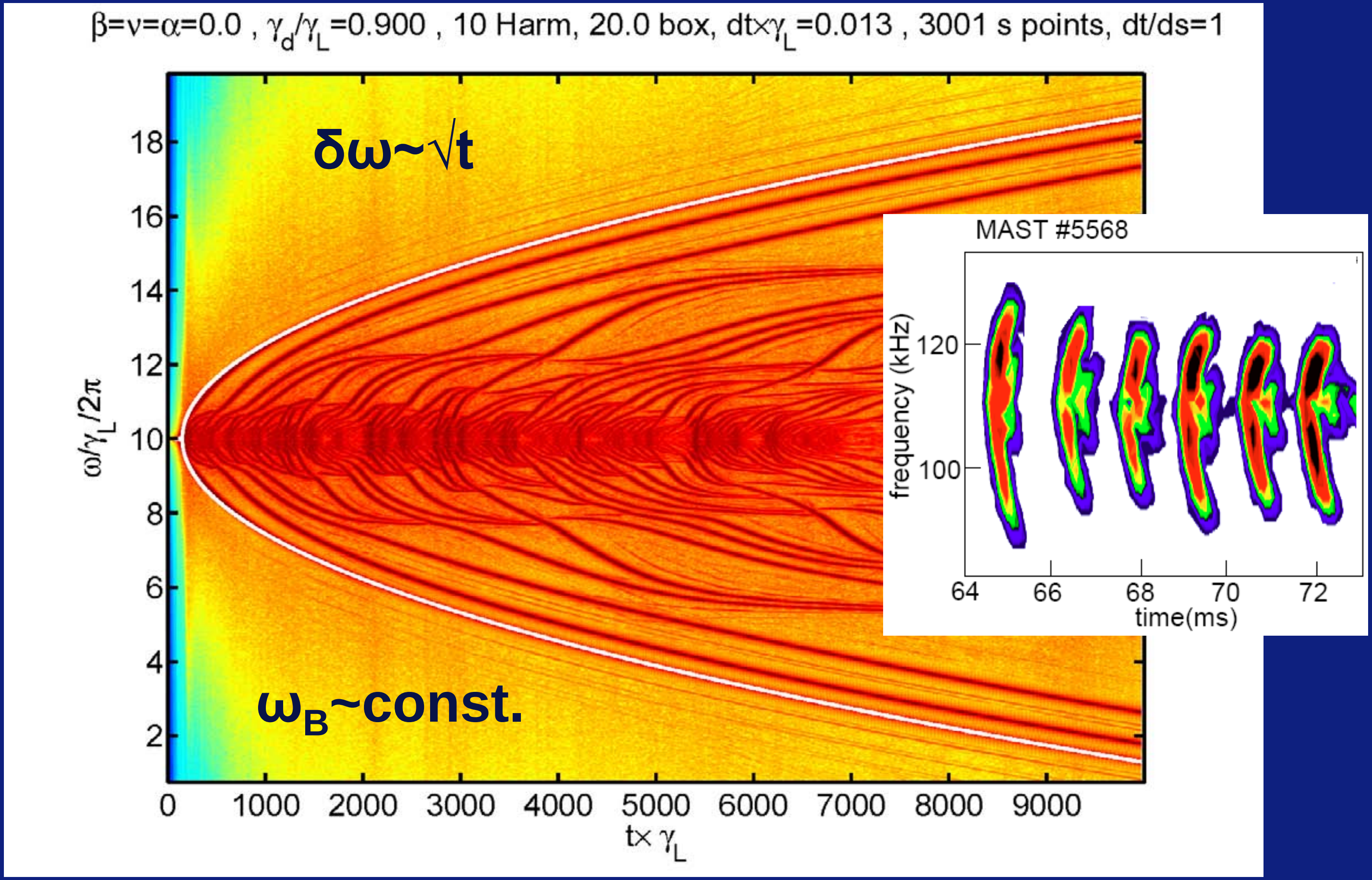


Frequency is changing for $\gamma_d / \gamma_L > 0.4$

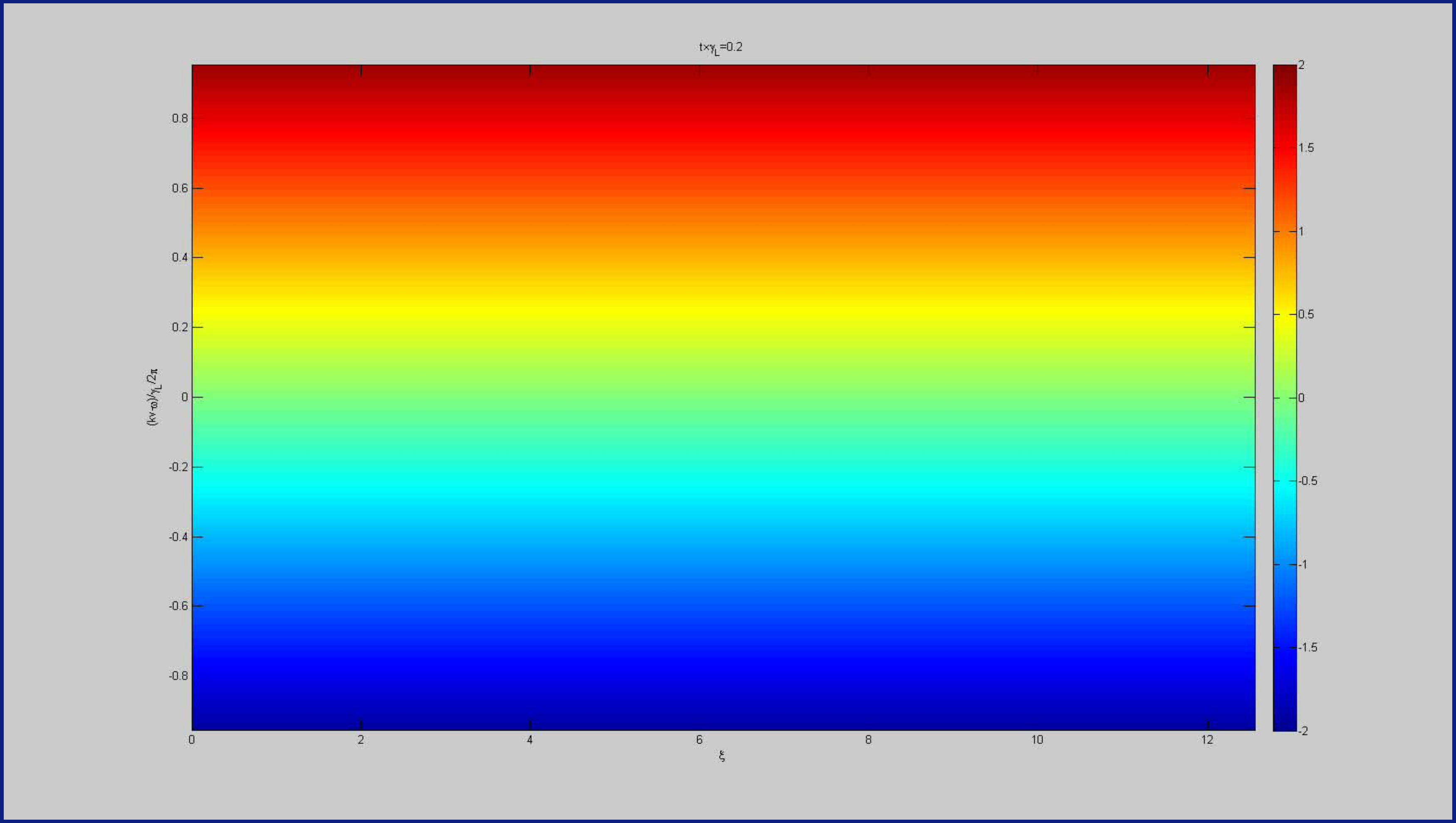
Marginal stability – Frequency chirping



Marginal stability – Frequency chirping



Marginal stability – Holes and clumps

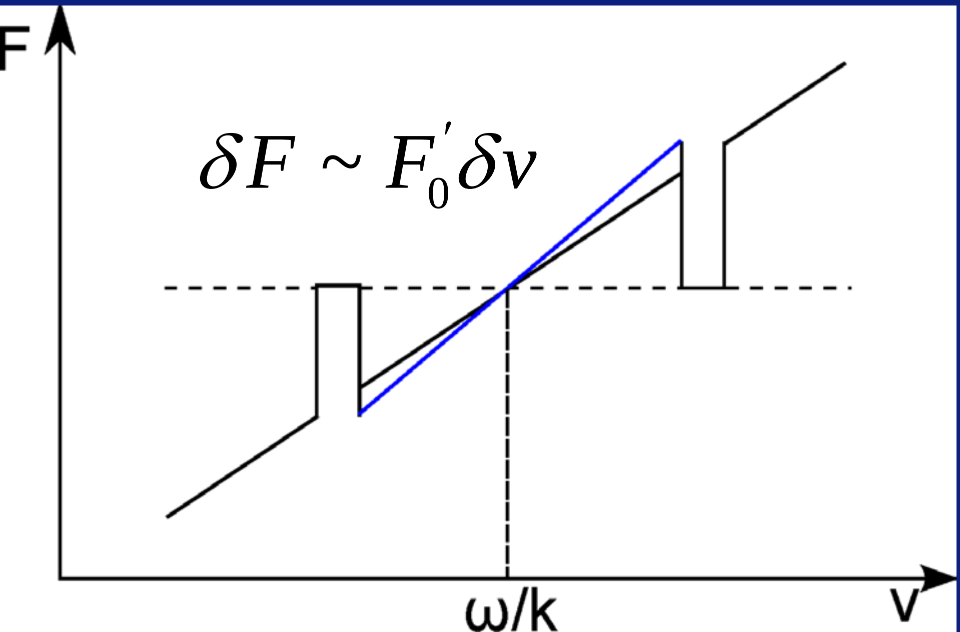


Spectral lines are holes and clumps in phase space

Marginal stability – Hole/clump dynamics

- Holes/clumps are the original resonant particles
- They are modulated beams/anti-beams→large effect even with small density, since $\omega = k\mathrm{v}_{\mathrm{b}}$

$$\varepsilon \approx 1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pb}^2}{(\omega - k\mathrm{v}_{\mathrm{bc}})^2} + \frac{\omega_{pb}^2}{(\omega - k\mathrm{v}_{\mathrm{bh}})^2} = 0$$

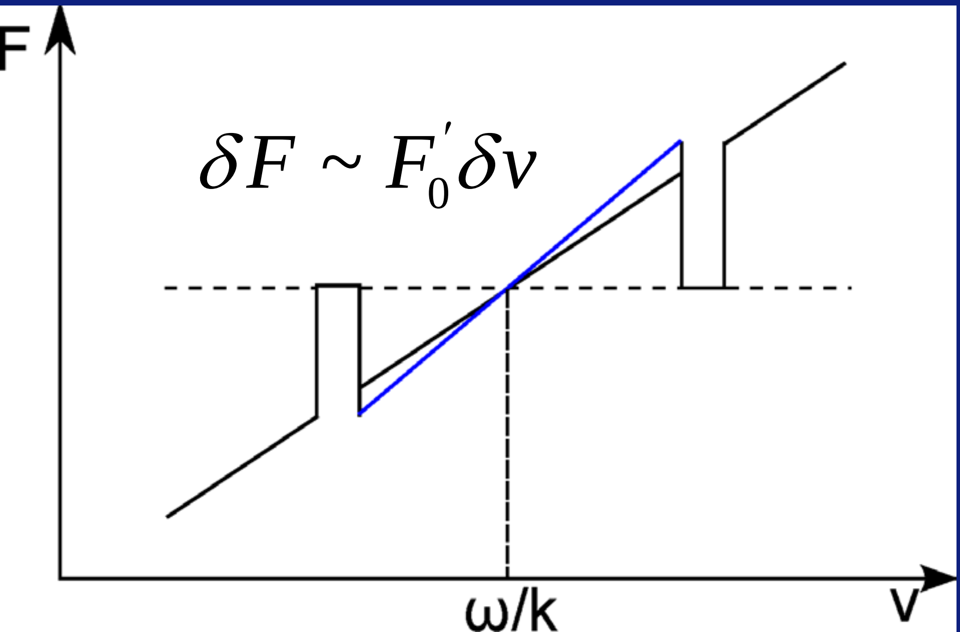


- Set $\omega = \omega_{pe}$ to lowest order. Correction from hole/clump will force other wave frequencies down/up→holes/clumps naturally move apart

Marginal stability – Hole/clump dynamics

- Holes/clumps are the original resonant particles
- They are modulated beams/anti-beams→large effect even with small density, since $\omega = k\mathrm{v}_{\mathrm{b}}$

$$\frac{2\delta\omega}{\omega_{pe}} = \frac{\omega_{pb}^2}{(\omega_{pe} - k\mathrm{v}_{\mathrm{bc}})^2} - \frac{\omega_{pb}^2}{(\omega_{pe} - k\mathrm{v}_{\mathrm{bh}})^2}$$

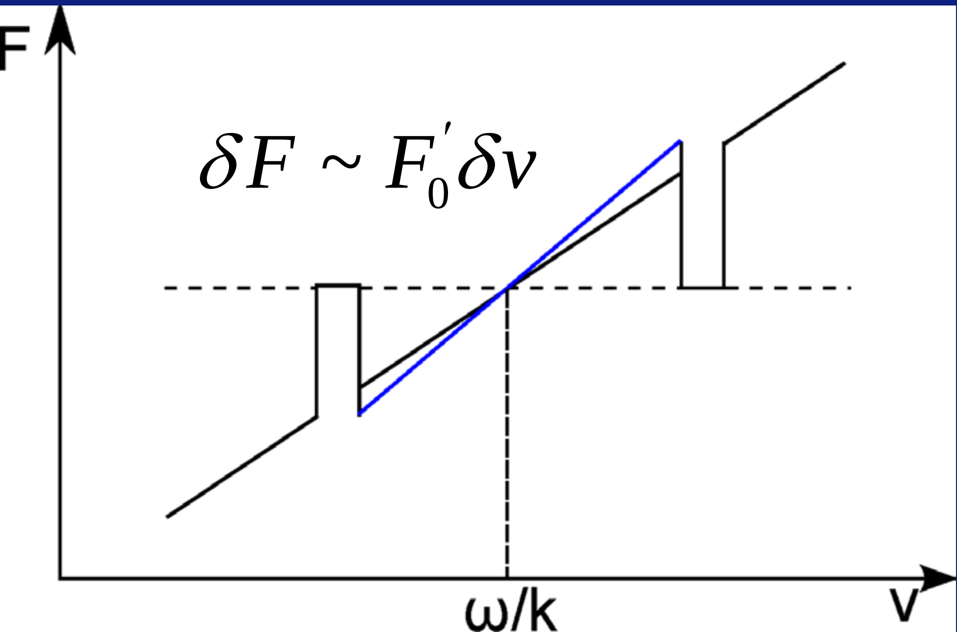


- Set $\omega = \omega_{pe}$ to lowest order. Correction from hole/clump will force other wave frequencies down/up→holes/clumps naturally move apart

Marginal stability – Hole/clump dynamics

- They move slowly compared to the bounce period
- Particles cant get inside separatrix→waterbag
- Trapped particles give most of δn_e

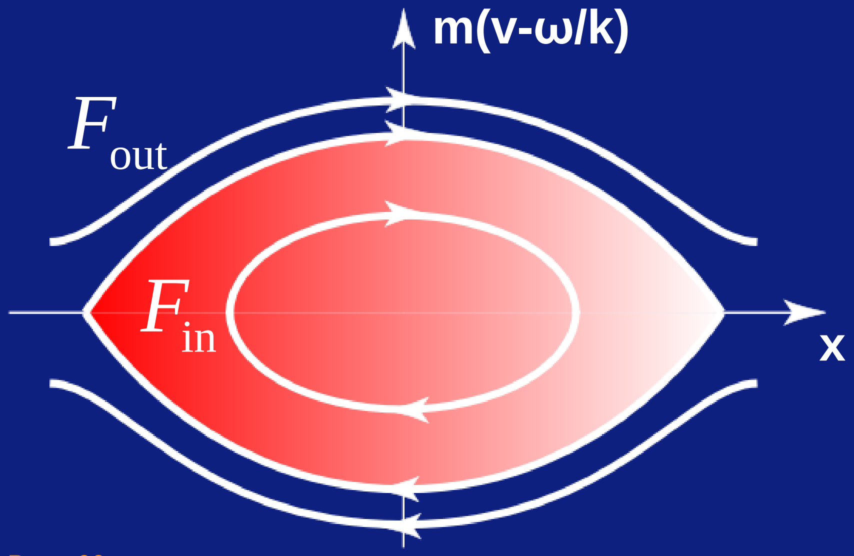
$$\nabla \cdot \varepsilon E = -|e|\delta n_e \rightarrow \delta\omega\omega_B^2 = \gamma_L \delta\omega\omega_B$$



- Works for $\omega \approx \omega_{pe}$
- → E is constant
- Hole or clump gets deeper/higher as it moves

Marginal stability – Hole/clump dynamics

- Hole: energy is required to move particles up ~ F_{in}
- Energy released as particles are forced over ~ F_{out}
- They must move to balance dissipation
- Deeper holes release more energy → √t chirp

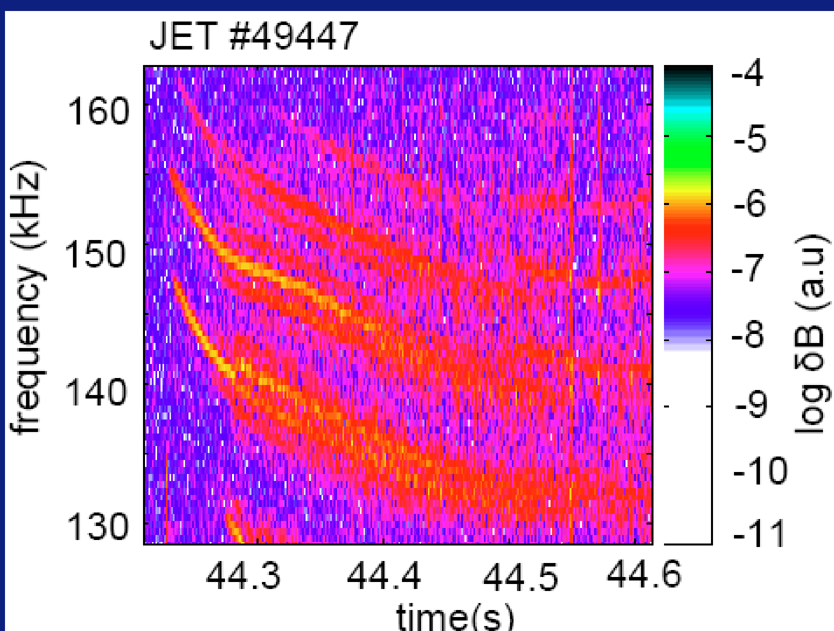


$$\underbrace{\gamma_d \omega_B^4}_{\text{dissipation}} = \underbrace{\gamma_L \delta\omega\omega_B \frac{\partial \delta\omega}{\partial t}}_{\text{power release}}$$

The Questions

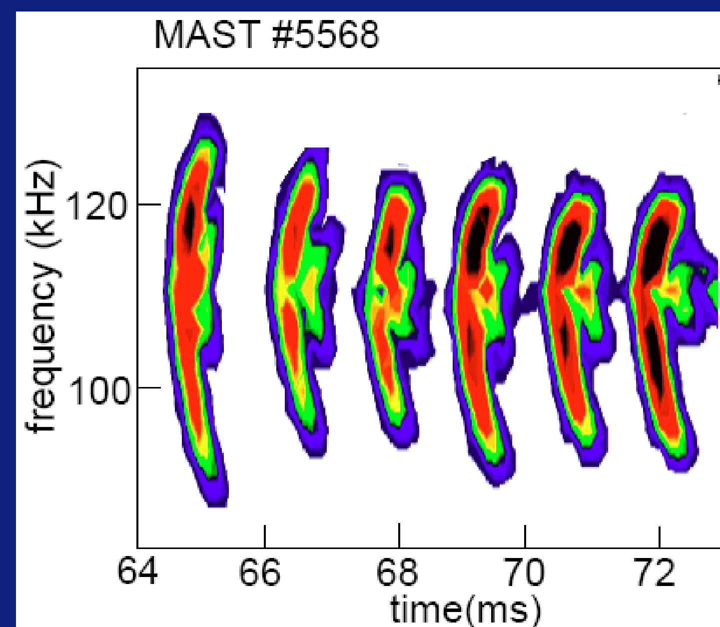
- How does a low density population produce a large effect
- How does the plasma produce such rich non linear evolution at different timescales
- How is it that the same modes driven by different particles look so different

ICRH drive (JET)



Heeter et.al PRL 85, 3177 (2000)

NBI drive (MAST)



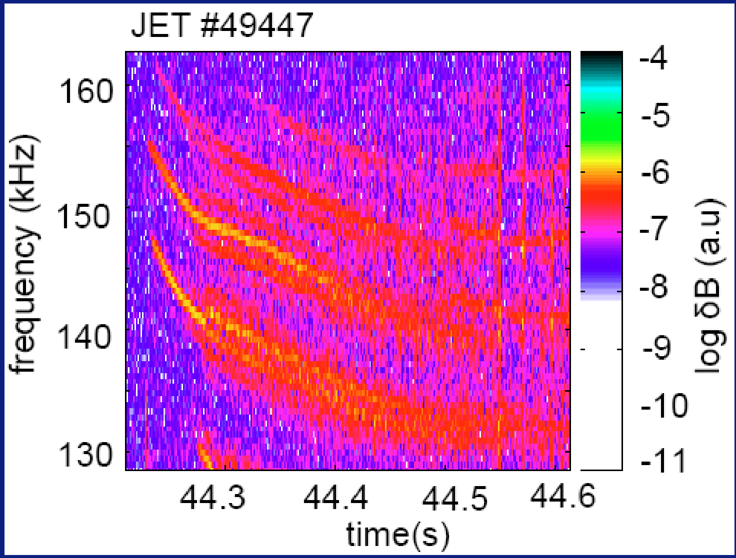
Pinches et.al PPCF, 46, S47 (2004)

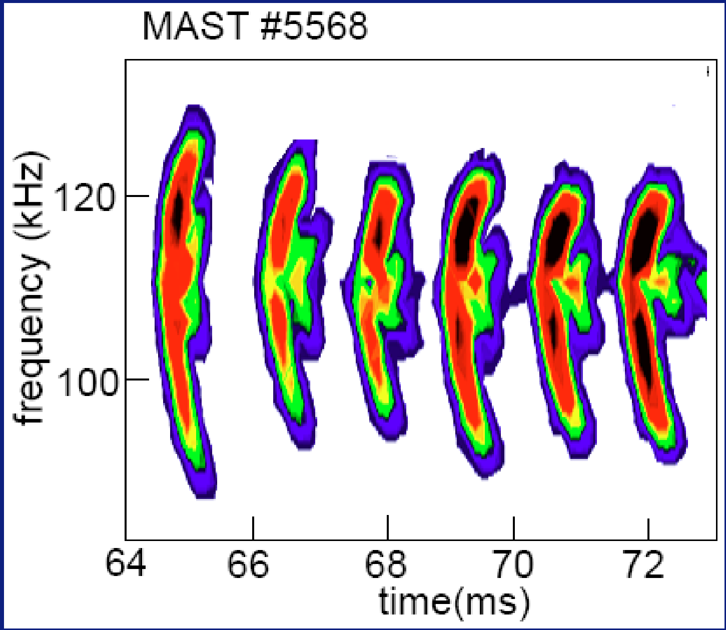
Collisionality - Revisited

- Collisionality not low enough to explain MAST
- NBI distribution determined by drag for $E \sim E_A >> E_{crit}$
- Dynamical friction (drag) collisions should be included

$$\left.\frac{dF}{dt}\right|_{coll} = \alpha^2 \left(\frac{\partial F}{\partial v} - \frac{\partial F_0}{\partial v} \right)$$

- Could this explain the bursting for beam driven TAEs





Mode evolution equation – Effect of drag

Near marginal stability the amplitude (*A*) of the unstable mode evolves according to the following equation

$$\frac{dA}{d\tau} = A(\tau) - \frac{1}{2}\int_0^{\tau/2} dz\, z^2 A(\tau - z) \int_0^{\tau-2z} dx\, e^{-\hat{\nu}^3 z^2 (2z/3+x) - \hat{\beta}(2z+x) + i\hat{\alpha}^2 z(z+x)} \times A(\tau - z - x) A^*(\tau - 2z - x)$$

$\hat{\nu}$ - Diffusion coefficient

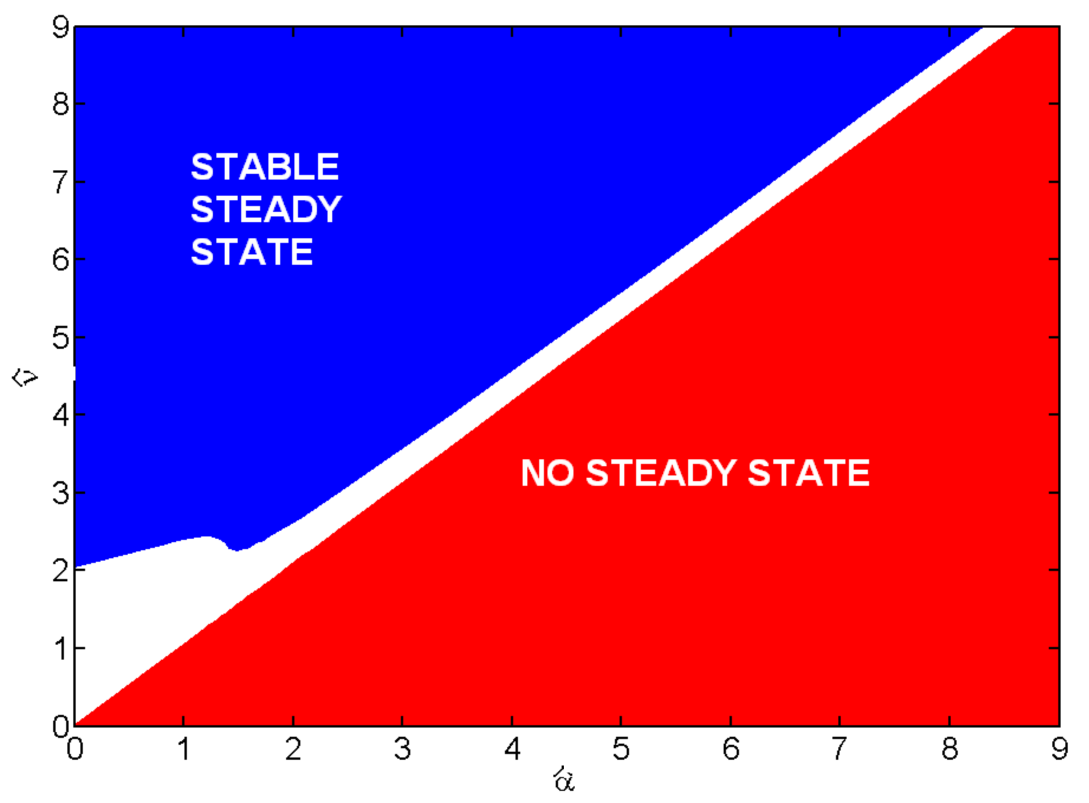
$\hat{\beta}$ - Krook coefficient

$\hat{\alpha}$ - Drag coefficient

- Drag gives oscillatory behaviour, in contrast to the Krook and diffusive cases.
- For drag – The oscillatory nature allows the sign to flip often → don't need low collisionality to get explosion

Marginal stability - Diffusion + drag

- For diffusion drag steady state solutions do exist
- For an appreciable amount of drag these solutions become unstable (pitch fork splitting etc.)
- Explosive solutions again when drag dominates

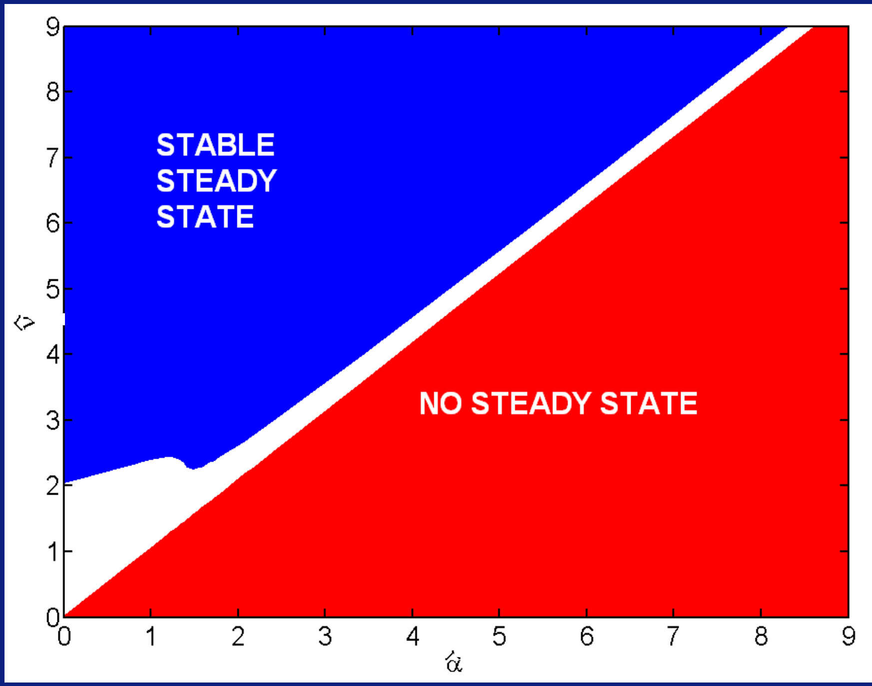


Lilley et.al PRL, 102, 195003 (2009)

Fully nonlinear drag regime - expectations

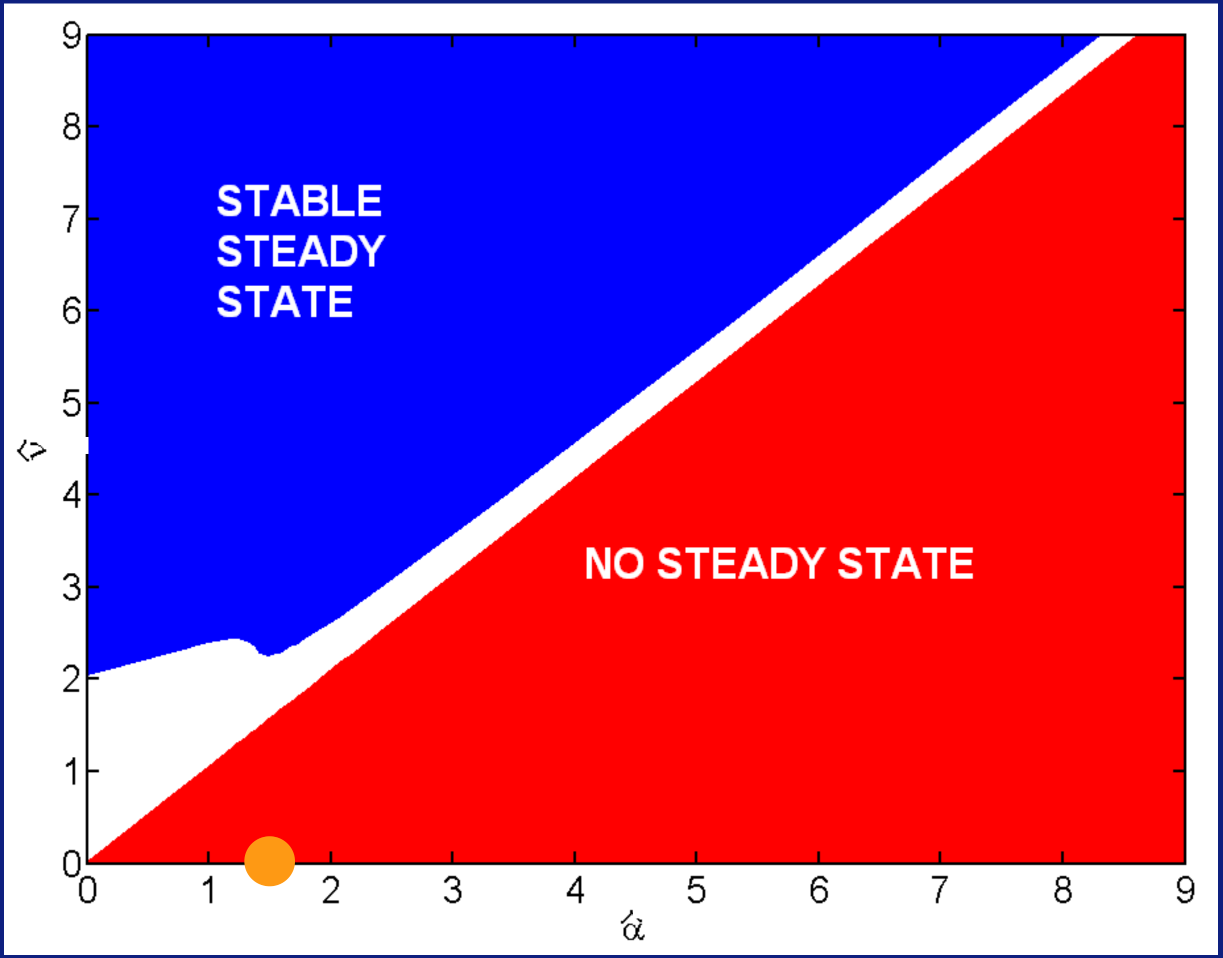
- Drag provides a preferred direction
- Expect asymmetry
- Holes move up in velocity
- Drag provides a flow, this

  acts like chirping
- Can drag replace chirping?

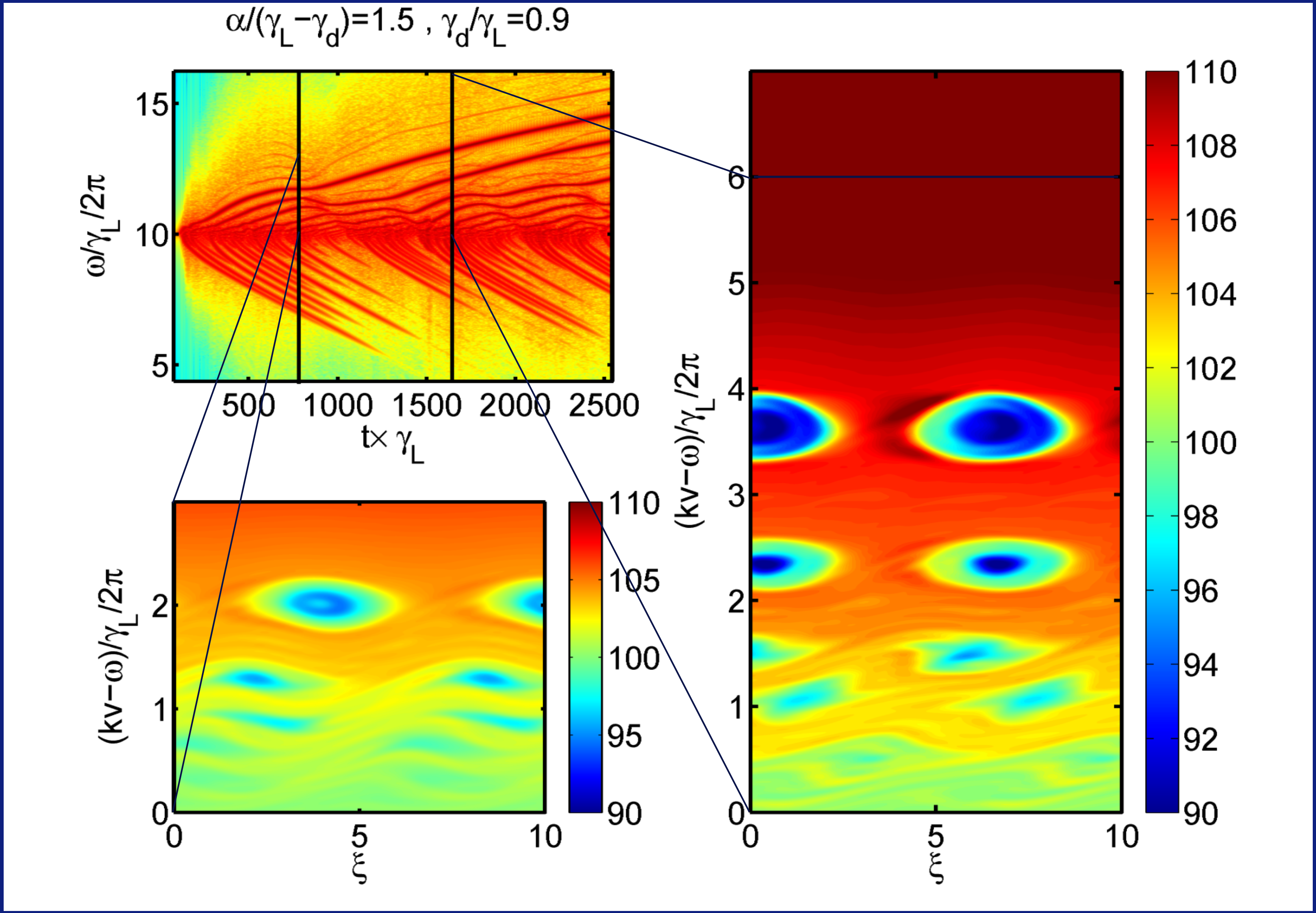


- i.e Can we get a steady state non-linear state away from the original resonance?

Pure drag



Pure Drag – Holes Grow Faster, Clumps Decay



Lilley et.al PoP, 17, 092305 (2010)

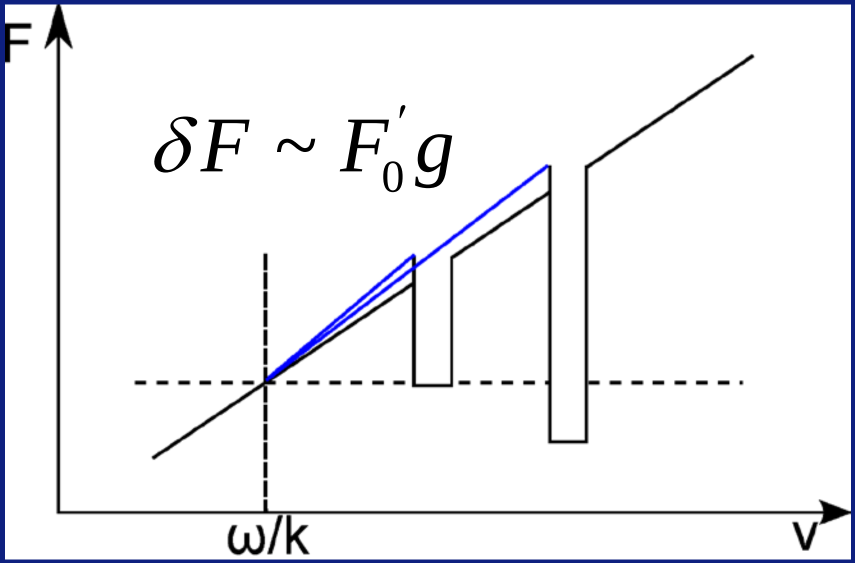
Pure drag – Growing holes

- Drag collision operator has a slowing down force and a sink

$$\left.\frac{dF}{dt}\right|_{\mathrm{coll}} = \frac{\alpha^2}{k}\left(\frac{\partial F}{\partial \mathrm{v}} - \frac{\partial F_0}{\partial \mathrm{v}}\right)$$

- Slowing down + sink returns distribution to equilibrium
- E field however can hold the hole in place working against slowing down force
- Sink still acts to lower F → deeper hole over time

Pure drag – Growing holes



- Deeper hole → bigger E

$$\delta\omega\omega_B^2 = \gamma_L \omega_B g$$

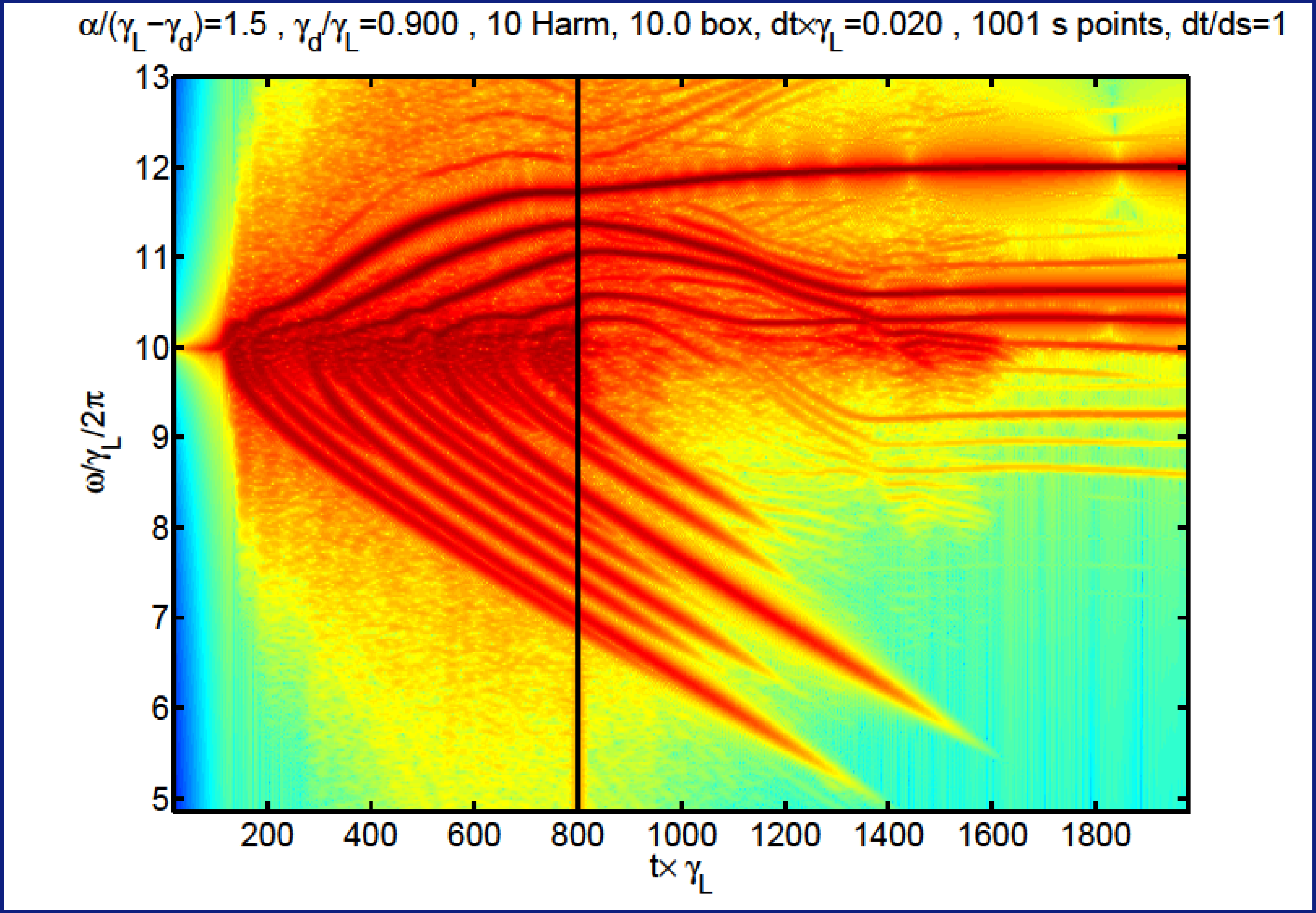
$$g = \delta\omega + \alpha^2 t$$

$$\gamma_d \omega_B^4 = \gamma_L g \omega_B \left(\frac{\partial \delta\omega}{\partial t} + \alpha^2 \right)$$

- When drag dominates no steady state is possible

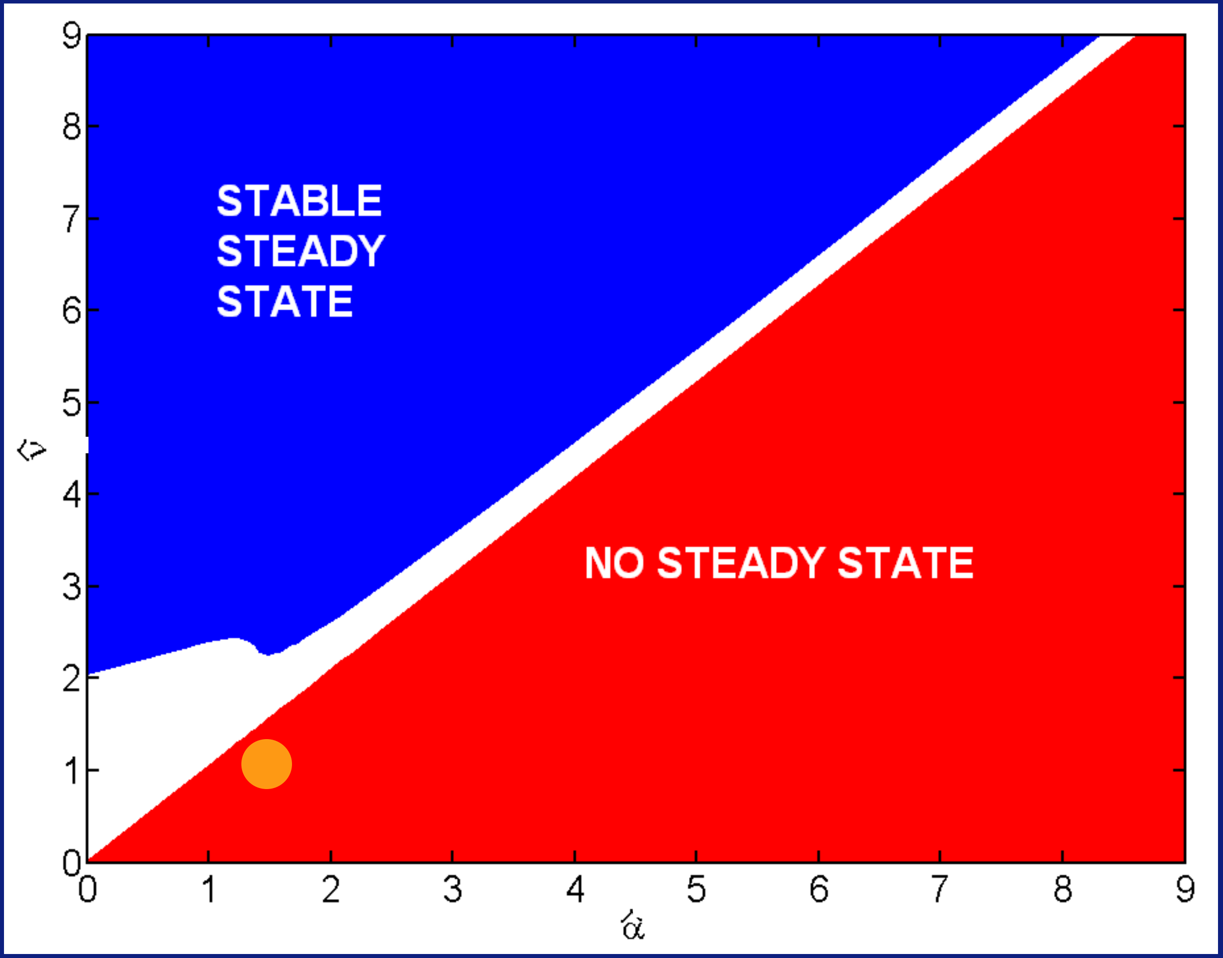
$$\omega_B \sim \alpha^{4/3} t^{1/3}, \quad \delta\omega \sim \gamma_L (\alpha t)^{2/3}$$

Pure drag – Saturation without slope

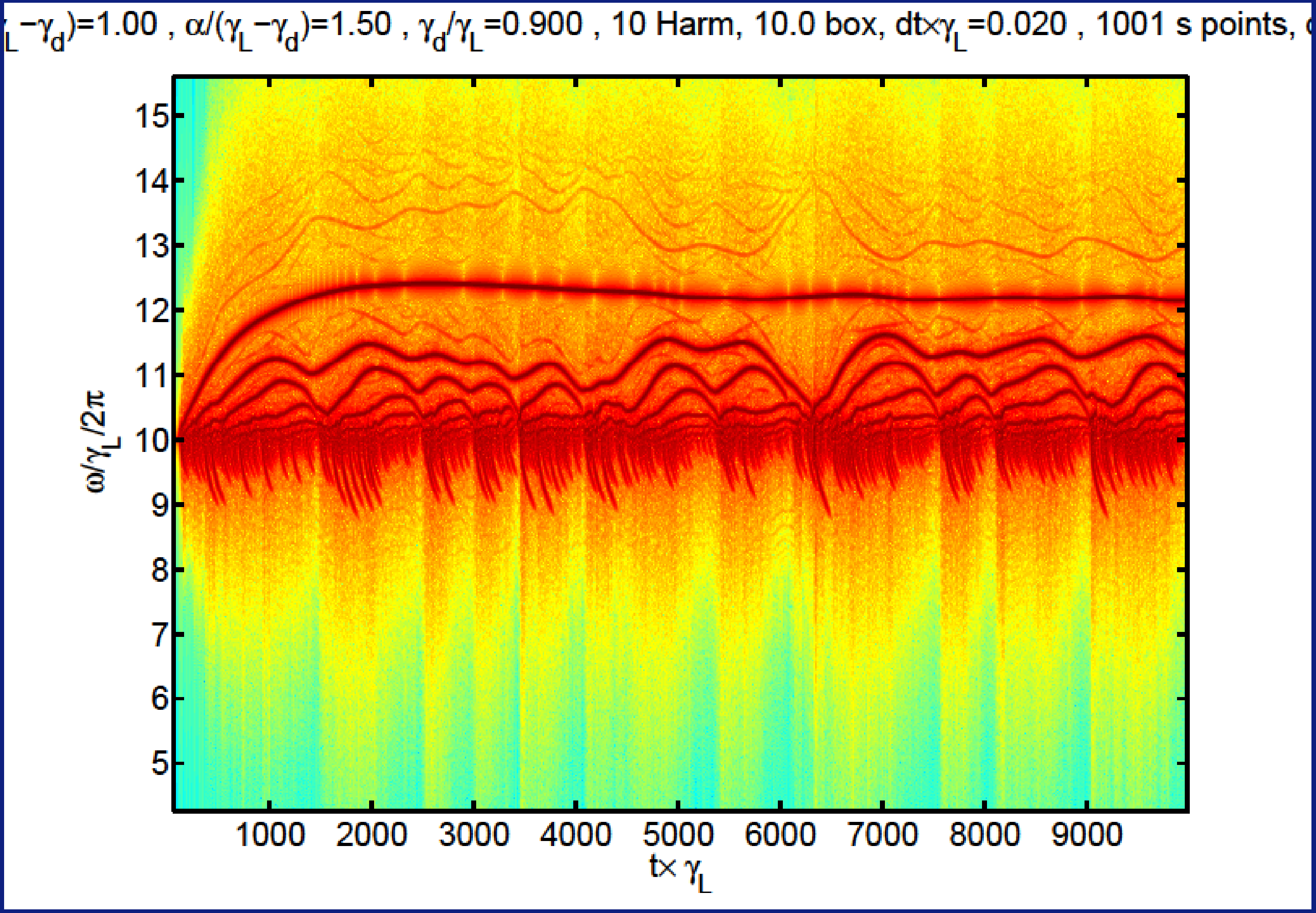


Remove slope and saturation is achieved

Now add some diffusion

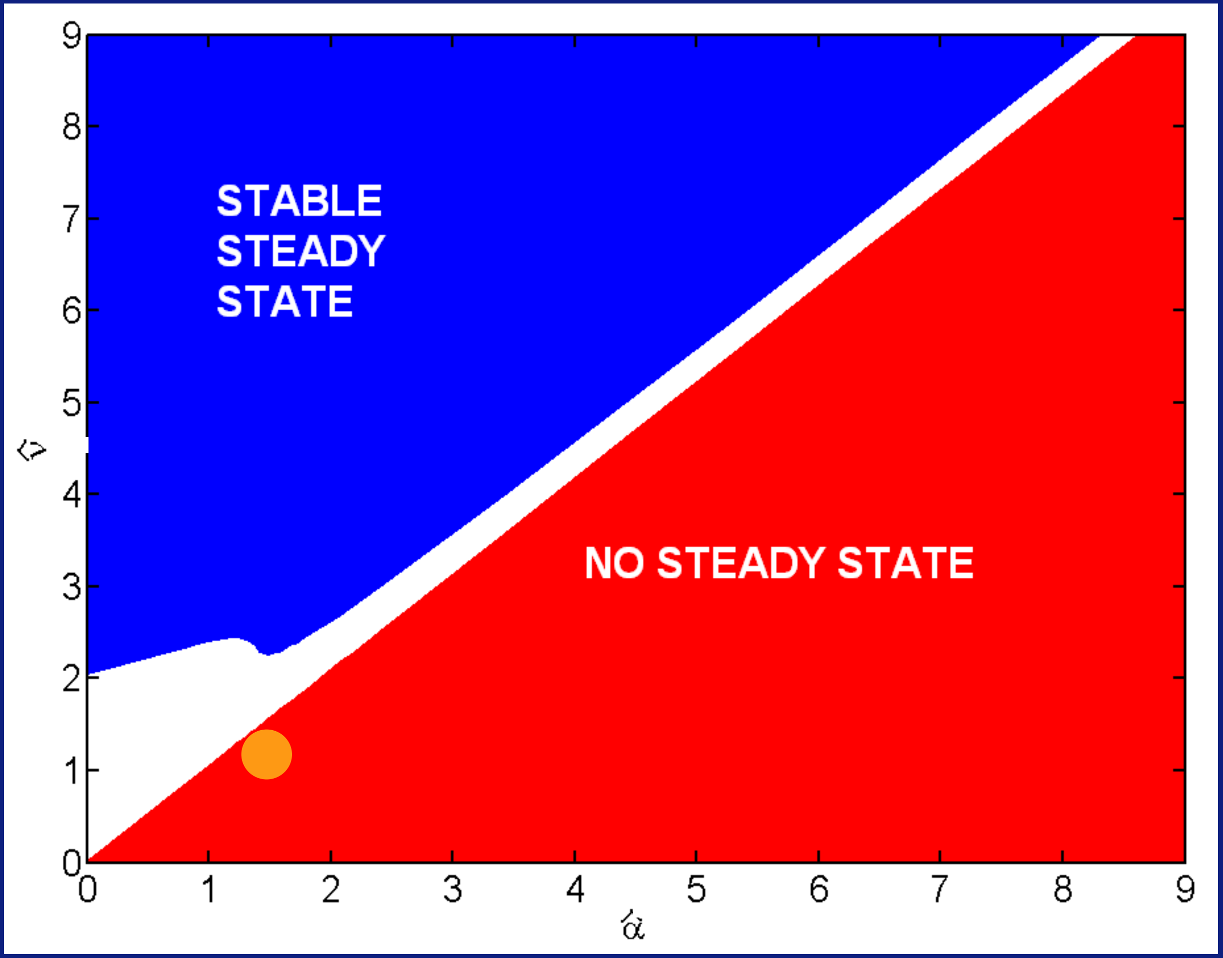


Drag + diffusion – Steady state hole

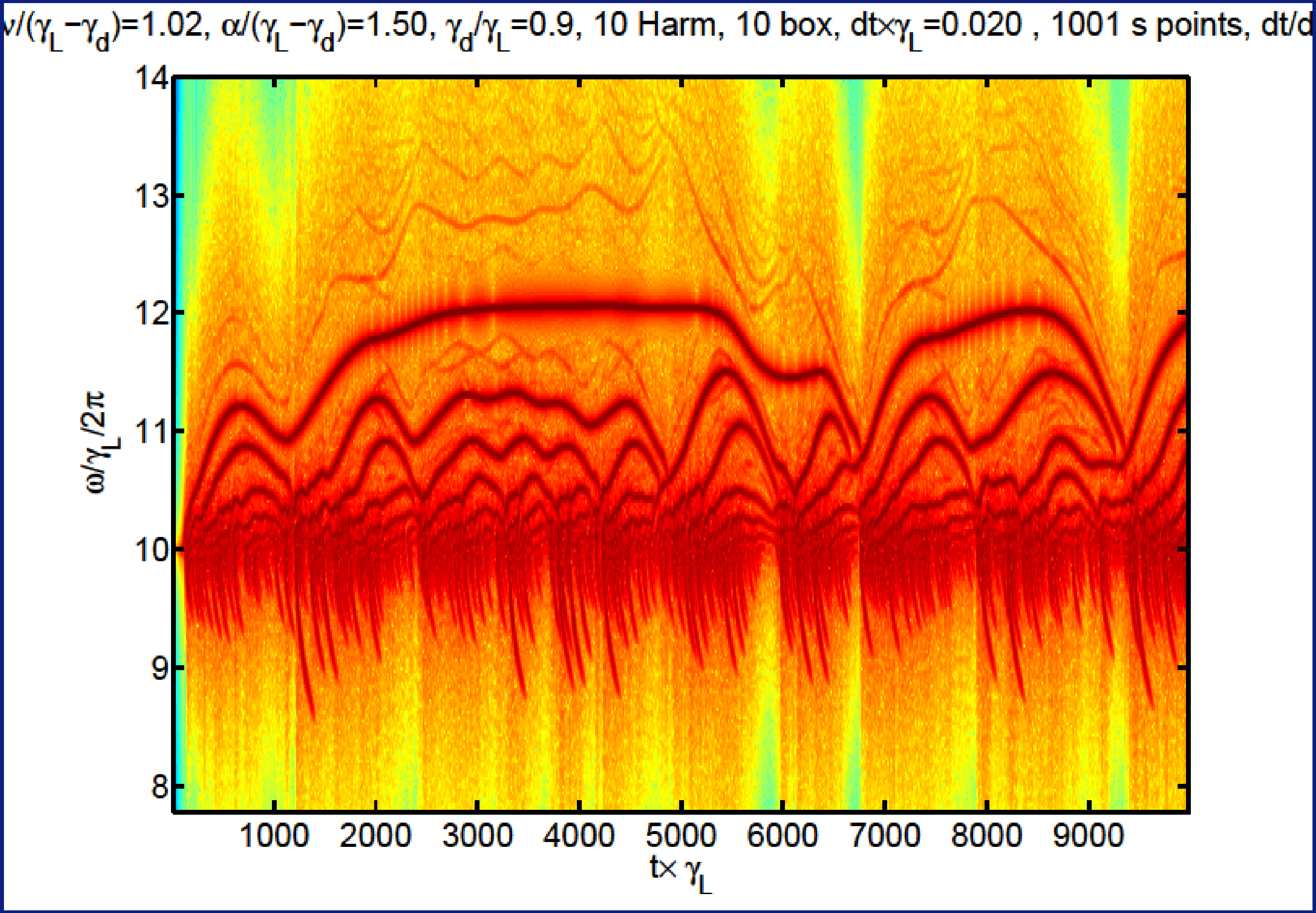


Lilley et.al PoP, 17, 092305 (2010)

Add a bit more diffusion

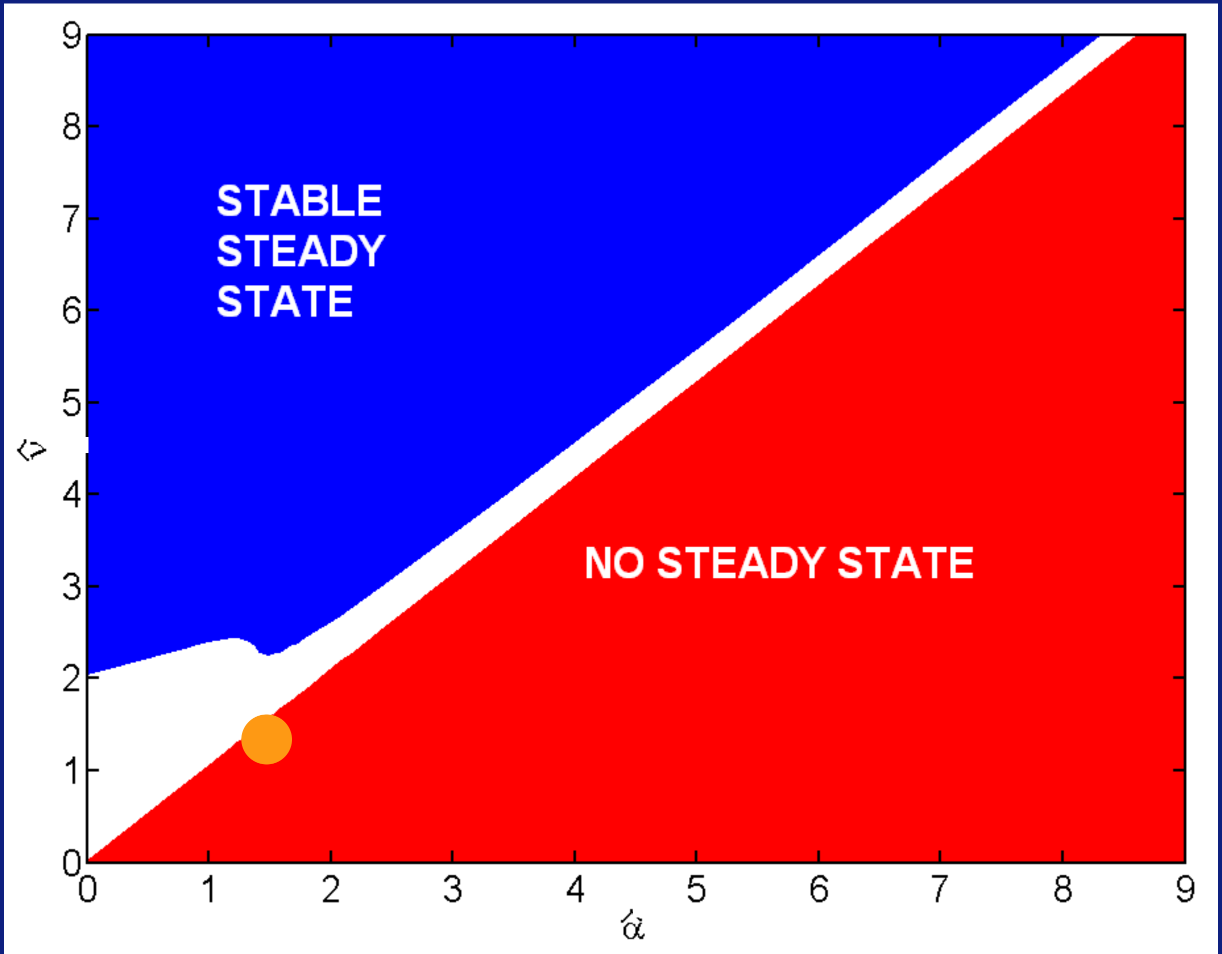


Drag + diffusion – Undulating frequency

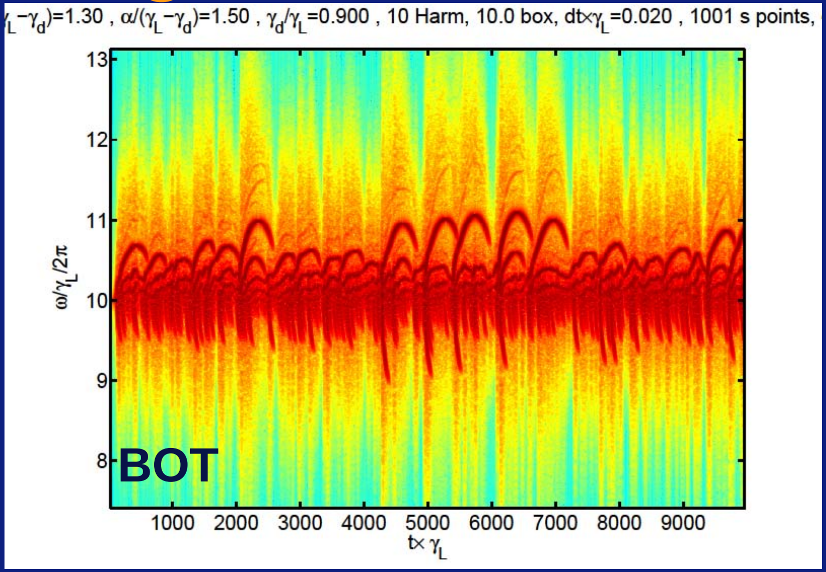


Lilley et.al PoP, 17, 092305 (2010)

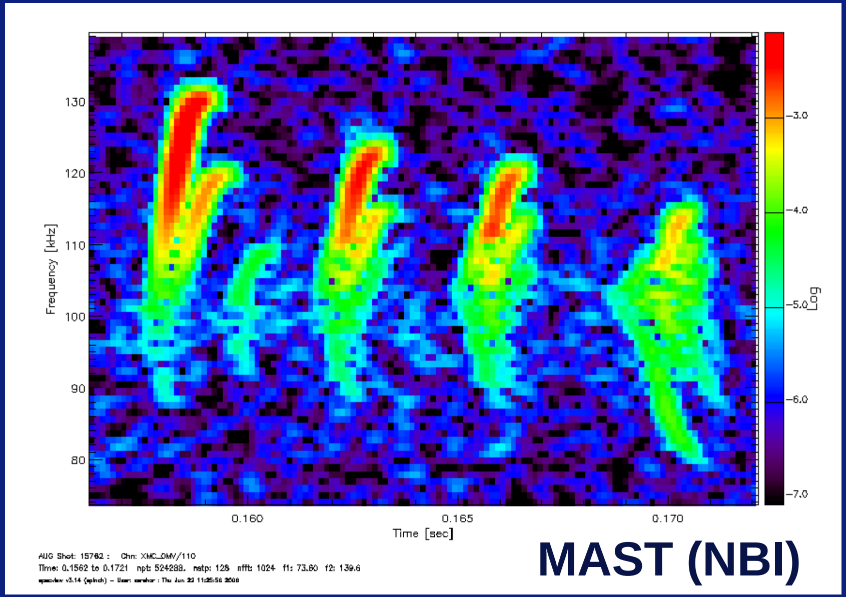
Keep adding diffusion

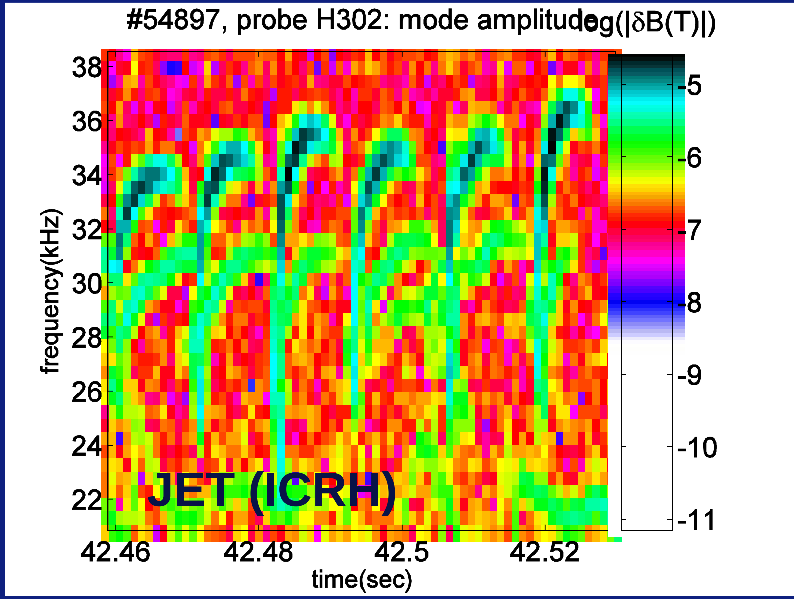


Drag + diffusion – Hooked frequency chirp

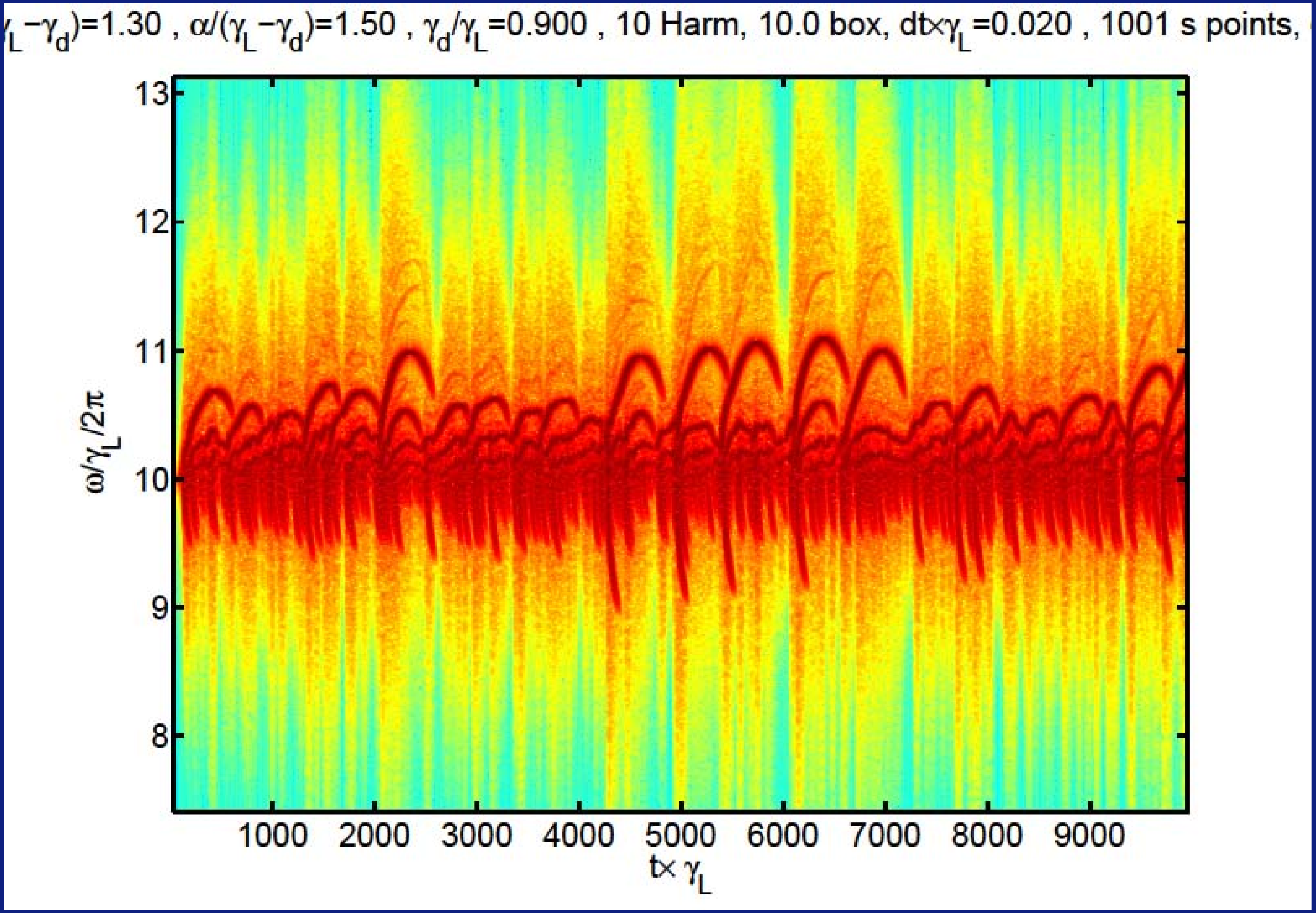


- Hooked frequency chirp seen in BOT
- Also seen in MAST (NBI) and JET (ICRH)





Drag + diffusion – Hooked frequency chirp



Hooks for the holes, clumps die sooner

Lilley et.al PoP, 17, 092305 (2010)

Drag Diffusion competition

$$\delta\omega\omega_B^2 = \gamma_L \omega_B g$$

- Poisson Equation

$$\frac{\partial g}{\partial t} + \frac{\nu^3}{\omega_B^2} g = \frac{\partial \delta\omega}{\partial t} + \alpha^2$$

- Diffusion fills, chirping and drag deepen

$$\gamma_d \omega_B^4 = \gamma_L g \omega_B \left(\frac{\partial \delta\omega}{\partial t} + \alpha^2 \right)$$

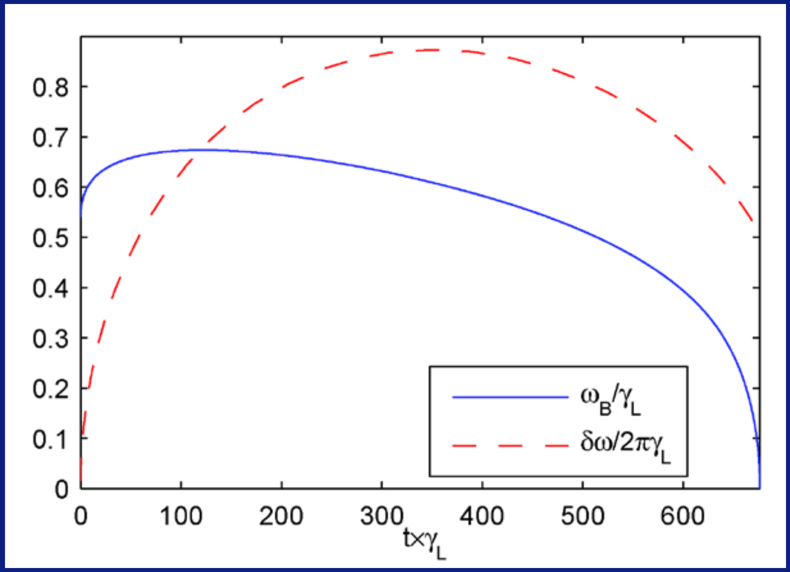
- Energy balance

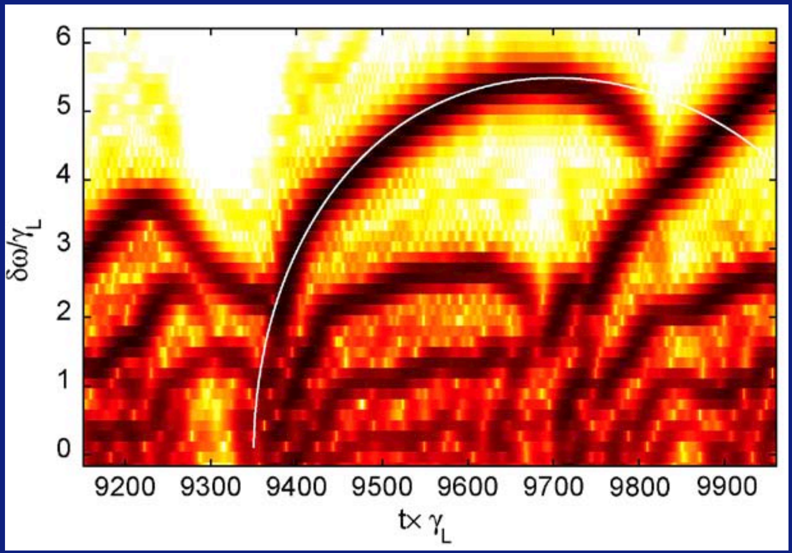
0-D Equations

$$x^2 = y\left(\frac{\partial y}{\partial \tau}+1\right)$$

$$a\frac{\partial(xy)}{\partial \tau}+\frac{y}{x}=\left(1+\frac{\partial y}{\partial \tau}\right)$$

- x=y=1 is steady state
- Unstable for a<1
- Stable for a>1





Lilley et.al PoP, 17, 092305 (2010)

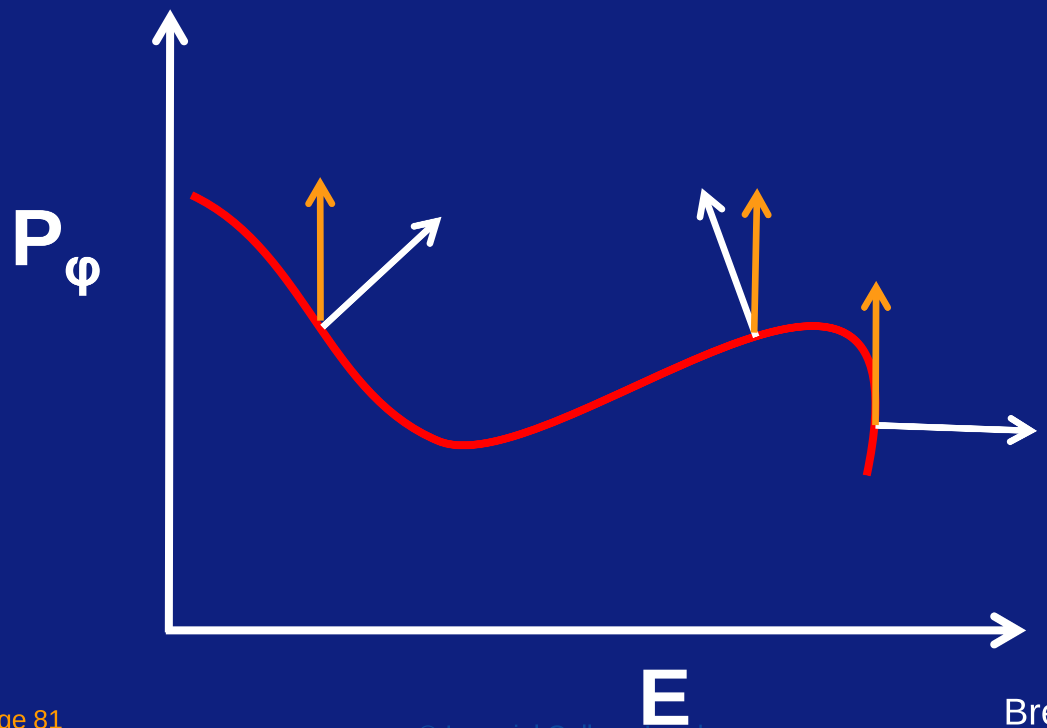
Generalisation to toroidal systems

Toroidal systems – A first glance (low freq.)

- Phase space resonance is more sophisticated

$$u \equiv k\mathrm{v} - \omega = 0 \;\rightarrow\; \Omega \equiv n\omega_{\varphi}\left(P_{\varphi}, E\right) - p\omega_{\theta}\left(P_{\varphi}, E\right) - \omega = 0$$

- Location of resonance varies



$$E - \frac{\omega}{n} p_{\varphi} = \mathrm{const.}$$

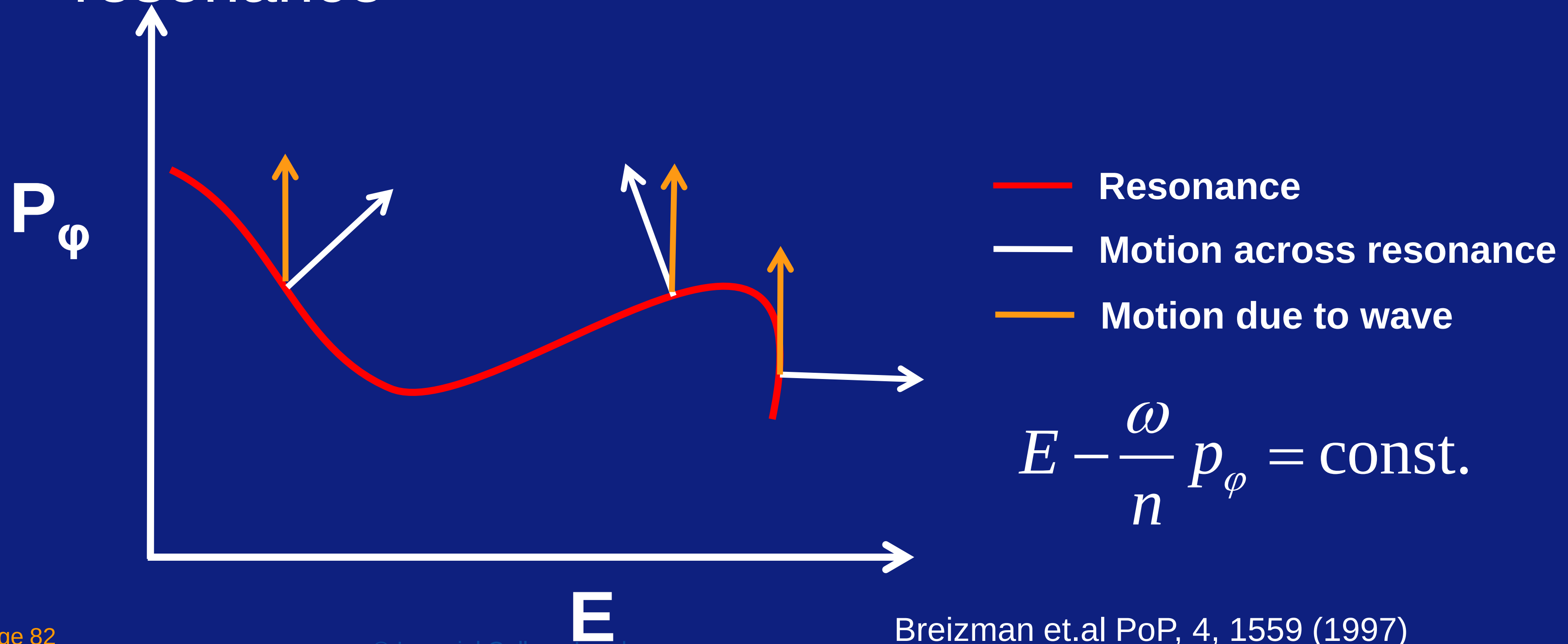
Breizman et.al PoP, 4, 1559 (1997)

Chirikov *Phys. Rep*. 52 263 (1979)

Toroidal systems – Reduction to 1-D model (low freq.)

- Particle motion along resonance does not lead to strange gradients in F, so neglect them

- Need projection of motion and collisions across resonance



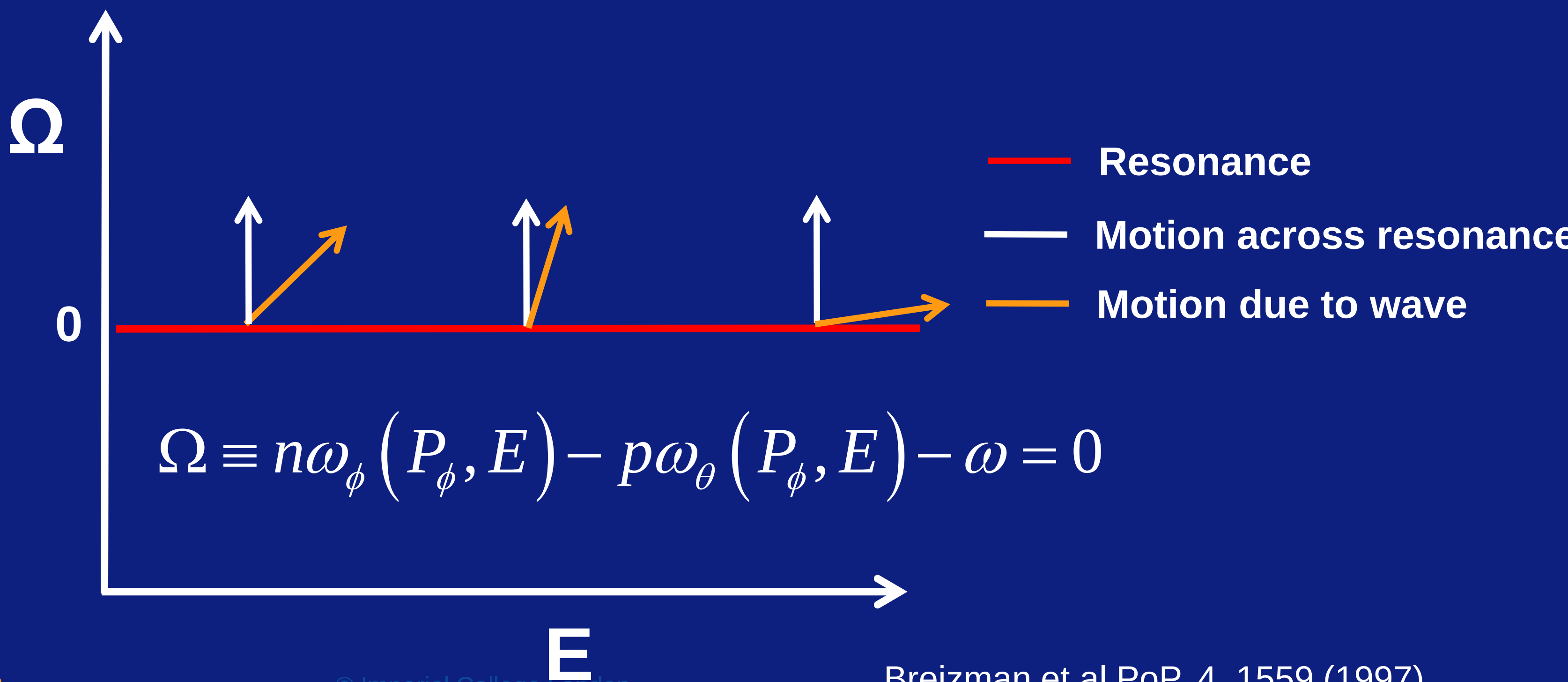
$$E - \frac{\omega}{n} p_\varphi = \text{const.}$$

Breizman et.al PoP, 4, 1559 (1997)

Chirikov *Phys. Rep.* 52 263 (1979)

Toroidal systems – Reduction to 1-D model (low freq.)

- Transform to coordinates that straighten resonance
- Motion across the resonance 1-D for given E and μ
- Must integrate over all E and μ to get the result



$$\Omega \equiv n\omega_{\phi}\left(P_{\phi},E\right)-p\omega_{\theta}\left(P_{\phi},E\right)-\omega=0$$

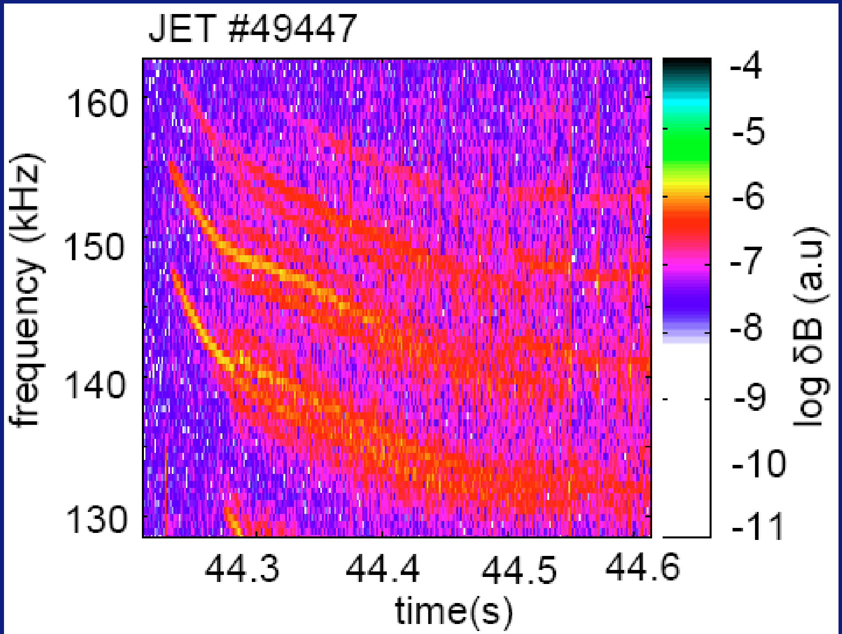
Breizman et.al PoP, 4, 1559 (1997)

Chirikov *Phys.* Rep. 52 263 (1979)

The Questions

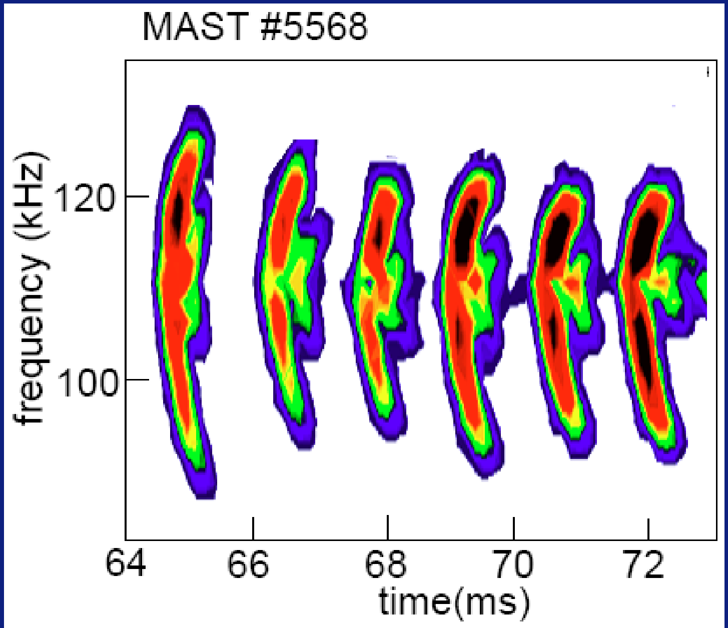
- How does a low density population produce a large effect
- How does the plasma produce such rich non linear evolution at different timescales
- How is it that the same modes driven by different particles look so different

ICRH drive (JET)



Heeter et.al PRL 85, 3177 (2000)

NBI drive (MAST)



Pinches et.al PPCF, 46, S47 (2004)

Experimental estimate for MAST and ITER

$$\frac{\nu_{\mathrm{TAE}}}{\alpha_{\mathrm{TAE}}} \sim \frac{T_e^{3/4} n_e^{1/6}}{B_0^{5/6}}$$

Drag vs diffusion depends on plasma parameters

$$\frac{\nu_{\mathrm{TAE}}}{\alpha_{\mathrm{TAE}}} \approx 0.2 - 1.6$$

MAST - beams - Drag can dominate → explosive

$$\frac{\nu_{\mathrm{TAE}}}{\alpha_{\mathrm{TAE}}} \approx 1.4$$

ITER – alphas – drag and diffusion comparable

Lilley et.al PRL, 102, 195003 (2009)

Remaining tasks...not exhaustive!

- 1D model: Can we understand more about why marginal stability seeds holes & clumps
- 1D model: Extending to high frequency involving cyclotron resonance
- 3D world: Put drag into fully toroidal codes to look at TAEs – now being done in HAGIS
- 3D world: Experimentally scan parameter space – needed to predict ITER operation

Conclusions

- Waves are important
- Resonance empowers the fast particles
- Marginal stability produces surprising route to energetic particle modes
- Plenty of non linear scenarios enriched by collisions
- Drag provides destabilising effect and gives an important observed asymmetry
- 1D model is good: for single resonance only

BOT Code

- Email bumpontail@gmail.com from your work email
- Please give your name, institution and your position
- It is free for you to use, modify and also distribute, but I encourage others to contact me for the code so that I can send updates as they become available